For all questions below, the answer (E) NOTA means "None of these answers".

1. Find the circumradius of a triangle with side lengths 14, 48, 50.

(A) 24 **(B)** 25 **(C)** 26 **(D)** 27 **(E)** NOTA

2. A circle is inscribed in a square that is inscribed in a circle that is inscribed in a square of side length 8. Find the area of the smaller circle.

(A) 2π **(B)** 4π **(C)** 8π **(D)** 16π **(E)** NOTA

3. Two supplementary angles measure $(2x + 76)^\circ$ and $(9x + 49)^\circ$. Find the measure of the larger angle.

(A) 64° **(B)** 92° **(C)** 94° **(D)** 116° **(E)** NOTA

4. Let ABC be a triangle with $m\angle A = 90^\circ$. If $AC = 105$ and $BC = 137$, find the perimeter of ABC.

(A) 324 **(B)** 330 **(C)** 334 **(D)** 340 **(E)** NOTA

5. Find the smallest possible value of an angle in a convex dodecagon whose angles form an arithmetic sequence of integers.

(A) 122° **(B)** 124° **(C)** 126° **(D)** 128° **(E)** NOTA

6. Let ABC be a triangle with $AB = 5$, $BC = 12$, $CA = 13$. Furthermore, let M be the midpoint of $\overline{\text{AC}}$ and N be the foot of the B altitude on side $\overline{\text{AC}}$. If the length of MN can be written as $\frac{p}{q}$, where p and q are relatively prime, find $p + q$.

(A) 132 **(B)** 137 **(C)** 145 **(D)** 151 **(E)** NOTA

7. Suppose a cube and a sphere have the same surface area. Find the ratio of the side length of the cube to the diameter of the sphere. *Note*: Answer choices are not rationalized.

(A) $\sqrt{\frac{\pi}{6}}$ **(B)** $\sqrt{\frac{\pi}{2}}$ **(C)** $\sqrt{\frac{2\pi}{3}}$ **(D)** $\sqrt{\pi}$ **(E)** NOTA

8. Let $ABCD$ be a unit square. A semicircle with diameter $\overline{\text{AB}}$ is drawn so that it lies outside of the square. If E is the midpoint of arc AB of the semicircle, what is the area of triangle CDE?

(A) $\frac{4}{5}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{1}{2}$ **(E)** NOTA

9. Three non-overlapping equilateral triangles with side length 1 are joined along their edges to form an isosceles trapezoid. What is the length of one of its diagonals?

(A) $\frac{2\sqrt{3}}{3}$ **(B)** $\frac{3}{2}$ **(C)** $\sqrt{3}$ **(D)** $\frac{4\sqrt{3}}{3}$ **(E)** NOTA

10. A right triangle with legs of length 7 and 24 is revolved around the side with length 24. Find the volume of the resulting figure.

(A) 336π **(B)** 364π **(C)** 392π **(D)** 420π **(E)** NOTA

11. Find the inradius of a triangle with side lengths 15, 18, and 21.

(A) $2\sqrt{6}$ **(B)** $3\sqrt{3}$ **(C)** 6 **(D)** $4\sqrt{3}$ **(E)** NOTA

12. Jeffrey and his friends arrive at the basketball court, but they realize that no one brought a ball. Luckily, Jeffrey finds a truncated icosidodecahedron nearby that can be used as a substitute. Given that it has 120 vertices and 180 edges, find the sum of the digits of the total number of faces on the "ball".

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** NOTA

13. Find the area of the trapezoid with bases lengths 17 and 42 and leg lengths 15 and 20.

(A) 295 **(B)** 354 **(C)** 413 **(D)** 472 **(E)** NOTA

14. Find the number of sides of a regular polygon whose exterior angles each measure 0.75°.

(A) 270 **(B)** 360 **(C)** 480 **(D)** 520 **(E)** NOTA

15. Let ω be a circle centered at O. Let X and Y lie on the circumference of ω and let P lie on $\overline{XY}$. If $PX = 5$, $PO = 3$, and the radius of ω is 7, find PY.

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** NOTA

16. Let $ABCD$ be a kite with perpendicular diagonals such that $m\angle BAC = 20^\circ$, $m\angle CBD = 45^\circ$, and $m\angle DCA = 70^\circ$. Find $m\angle CDA$.

(A) 60° **(B)** 65° **(C)** 70° **(D)** 75° **(E)** NOTA

17. In triangle ABC, $AC = 7$. D lies on $\overline{\text{AB}}$ such that $AD = BD = CD = 5$. Find BC.

(A) $2\sqrt{6}$ **(B)** 12 **(C)** $\sqrt{51}$ **(D)** 13 **(E)** NOTA

18. Consider a right rectangular prism with side lengths x, y, z. Suppose that its volume is 300, its surface area is 295, and the sum of the lengths of all of its edges is 85. Find the volume of a right rectangular prism with side lengths $x + 2, y + 2, z + 2$.

(A) 688 **(B)** 689 **(C)** 690 **(D)** 691 **(E)** NOTA

19. It's 1:50, which means school's out for Sri! However, being the avid geometer he is, he decides to remain at school until the first moment when the hour and minute hand on his watch form a 90° angle. How many minutes does Sri wait?

(A) 4 **(B)** $\frac{46}{11}$ **(C)** $\frac{48}{11}$ **(D)** $\frac{50}{11}$ **(E)** NOTA

20. Let $A = (0,0)$, $B = (13,-2)$, and $C = (7,4)$. Find the number of lattice points contained in the interior of triangle ABC. Note: (x,y) is a lattice point if x and y are both integers.

(A) 29 **(B)** 30 **(C)** 31 **(D)** 32 **(E)** NOTA

21. A point P lies in the interior of equilateral triangle ABC such that the distances from P to AB, BC, and CA are 3, 4, and 5, respectively. Find the area of ABC.

(A) $12\sqrt{3}$ **(B)** $24\sqrt{3}$ **(C)** $36\sqrt{3}$ **(D)** $48\sqrt{3}$ **(E)** NOTA

22. Find the sum of the absolute values of the x and y intercepts of the line passing through the intersection points of $(x+2)^2+(y-6)^2=25$ and $(x-4)^2+(y-3)^2=5$.

(A) 1 **(B)** $\frac{5}{4}$ **(C)** $\frac{8}{3}$ **(D)** $\frac{3}{2}$ **(E)** NOTA

23. Let ABC be a triangle with $AB = 13$, $BC = 14$, $CA = 15$. Let $PQRS$ be a rectangle inscribed in ABC such that P and Q lie on BC, R lies on CA, and S lies on AB. Find the maximum possible area of $PQRS$.

(A) 35 **(B)** 42 **(C)** 56 **(D)** 63 **(E)** NOTA

24. The coordinates of three vertices of a parallelogram are $(1,1)$, $(2,4)$, and $(-5,1)$. Compute the area of the parallelogram.

(A) 6 **(B)** 9 **(C)** 18 **(D)** 36 **(E)** NOTA

25. Consider a semi-circle with diameter $\overline{\text{AB}}$. Let points C and D be on diameter $\overline{\text{AB}}$ such that $\overline{\text{CD}}$ forms the base of a square inscribed in the semicircle. Given that $CD = 2$, compute the length of AB.

(A) $\sqrt{5}$ **(B)** $\sqrt{10}$ **(C)** $2\sqrt{5}$ **(D)** $2\sqrt{10}$ **(E)** NOTA

26. Let $ABCD$ be a rectangle with $AB = 6$ and $BC = 10$. Let K, L lie on side $\overline{\text{AB}}$ such that $AK = KL = LB$ and let M, N lie on side $\overline{\text{CD}}$ such that $CM = MN = ND$. Find area of the region formed by the union of parallelograms $AKCM$ and $BLDN$.

(A) 35 **(B)** 25 **(C)** 15 **(D)** 5 **(E)** NOTA

27. Let ABC be a triangle with heights 7, 10, 14. If the perimeter of ABC can be expressed as $\frac{p\sqrt{q}}{r}$, where p, q, r are positive integers with p and r relatively prime, and $\sqrt{q}$ is in simplest radical form, then find $p+q+r$.

(A) 85 **(B)** 89 **(C)** 97 **(D)** 107 **(E)** NOTA

28. A triangle has all integral side lengths and two sides that measure 6 and 8. What is the sum of all possible lengths for the third side?

(A) 78 **(B)** 88 **(C)** 89 **(D)** 91 **(E)** NOTA

29. Let V be the number of vertices an icosahedron has. Let E be the number of edges an icosahedron has. What is $E - V$?

(A) 10 **(B)** 12 **(C)** 16 **(D)** 18 **(E)** NOTA

30. Let ABC be a triangle such that $BC = 6$, $AB + AC = 10$, and $\cos B + \cos C = 1$. If the area of ABC can be expressed as $\frac{p\sqrt{q}}{r}$, where p, q, r are positive integers with with p and r relatively prime, and $\sqrt{q}$ is in simplest radical form, then find $p + q + r$.

(A) 41 **(B)** 42 **(C)** 43 **(D)** 44 **(E)** NOTA