ANSWERS TO TESTS FOR GAITHER/LETO INVITATIONAL FEBRUARY 22, 1997

CALCULUS	PRECALC	ALG2	GEOM	ALG 1	INT SCHOOL
1. D	1. D	1. A	1. D	1. A	1. A
2. D	2. E	2. C	2. A	2. B	2. B
3. C	3. D	3. B	3. C	3. D	3. A
4. A	4. C	4. E	4. D	4. E	4. D
5. C	5. C	5. ~~D~~ A	5. D	5. C	5. C
6. B	6. B	6. A	6. C	6. D	6. D
7. E	7. E	7. B	7. A	7. C	7. D
8. C	8. B	8. B	8. C	8. C	8. B
9. A	9. B	9. E	9. D	9. E	9. E
10. C	10. D	10. C	10. D	10. A	10. C
11. B	11. C	11. A	11. B	11. C	11. C
12. A	12. A	12. A	12. A	12. D	12. C
13. C	13. A	13. A	13. C	13. B	13. A
14. D	14. D	14. C	14. B	14. D	14. B
15. A	15. D	15. E	15. C	15. D	15. A
16. B	16. A	16. A	16. C	16. C	16. C
17. B	17. A	17. A	17. D	17. A	17. D
18. E	18. A	18. C	18. ~~X~~ E	18. B	18. B
19. D	19. D	19. D	19. D	19. A	19. C
20. A	20. A	20. B	20. B	20. A	20. A
21. D	21. B	21. A	21. C	21. B	21. D
22. C	22. D	22. C	22. B	22. D	22. C
23. D	23. C	23. C	23. D	23. D	23. E
24. B	24. B	24. D	24. B	24. B	24. D
25. A	25. C	25. A	25. A	25. E	25. D
26. D	26. E	26. B	26. C	26. A	26. C
27. B	27. C	27. B	27. A	27. C	27. B
28. E	28. C	28. C	28. D	28. B	28. B
29. A	29. A	29. A	29. C	29. D	29. E
30. A	30. B	30. C	30. A	30. C	30. A
					31. B
					32. C
					33. A
					34. A
					35. D
					36. B
					37. B
					38. B
					39. D
					40. D
					41. A
					42. E
					43. C
					44. A
					45. C

ALPHA - INDIVIDUAL SOLUTIONS — Gather '97

1) $\tan^2\theta + 1 = \sec^2\theta$

$\cos\theta = \pm \frac{1}{\sqrt{\frac{12}{5}^2 + 1}}$

$= \pm 5/13$ QIII

$\therefore -5/13$ **D**

2) $\frac{3x^2 - x - 3}{x-2} = 3x + 5 + \frac{7}{x-2}$

SLANT: $y = 3x+5$

VERTICAL: $x = 2$ 2 **E**

3) m: in MAθ h: hard-boiled

e: egg GIVEN: $m \to h$, $h \to e$

Law of contraposition:

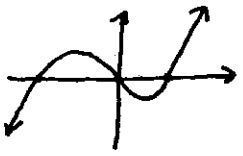
$\sim e \to \sim h$ ∧ $\sim h \to \sim m$

Law of syllogism:

$\sim e \to \sim m$ **D**

4) $y = x^3 + x^2 - 6x$

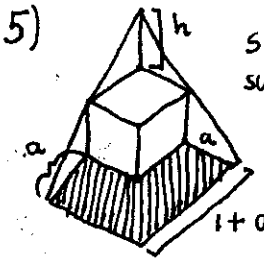
$= x(x+3)(x-2)$



$y \le 0$ iff

$x \in (-\infty, -3] \cup [0, 2]$ **C**

5)



$s = 1$

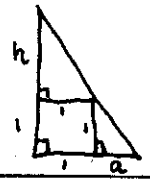
surf $A = 6$ $\quad 3 \cdot$ surf $A = 18$

$A_5 = 1\cdot a + (1+a)\cdot a$

$= a^2 + 2a = 18$

$= \frac{1}{h^2} + \frac{2}{h}$

$18h^2 - 2h - 1 = 0$



$\frac{a}{1} = \frac{1}{h}$ $\quad h = \frac{1+\sqrt{19}}{18}$ **C**

$a = \frac{1}{h}$

$1 + 19 - 18 = 2$

6) $\frac{1}{3} = .\overline{33}$ or $.\overline{99} = \sum_{n=1}^{\infty} \frac{9}{10^n}$

$3 \cdot 1/3 = .\overline{99} = 1$ $\quad = \frac{\frac{9}{10}}{1 - \frac{1}{10}} = 1$

$\lfloor 1 \rfloor = 1$ **B**

7) $n = m_1 + m_2$ $\quad m_1 = 3/4$ $\quad m_2 = -3/4$

$= 3/4 - 3/4$

$= 0$

$\frac{n}{m-p} = 0$ (if $m - p \neq 0$)

$\frac{x^2}{16} - \frac{y^2}{9} = 1$ $\quad p = \frac{c}{a} = \frac{\sqrt{7}}{3}$

$m = 2b = 6$

8) $710_n = 7n^2 + n$ $\quad 7n^2 + n = 576$

$7n^2 + n - 576 = 0$

$(7n + 64)(n - 9) = 0$

$n \neq -\frac{64}{7}$ $\quad n = 9$ **B**

9) $1 + \cot^2\theta = \csc^2\theta$ $\quad a = \csc\theta$

$\cot^2\theta - 3a + 3 = 0$

$a^2 - 3a + 2 = 0$ $\quad \sin\theta = \frac{1}{2}$ or 1

$(a-1)(a-2) = 0$ $\quad \theta \in \{\pi/6, \pi/2, 5\pi/6\}$

$\csc\theta = 1$ or 2 [illegible]

10) $(a+b)^n = a^n + n a^{n-1}b + \frac{n(n-1)}{2!} a^{n-2}$

$+ \frac{n(n-1)(n-2)}{3!} a^{n-3}b^3 + \dots$

$(1+1)^{1/2} = \sqrt{2} \approx 1^{\frac{1}{2}} + \frac{1}{2}\cdot 1^{-1/2}1^1 + \frac{-1}{8}\cdot 1$

$+ \frac{1}{16} 1^{-5/2} 1^3 - \frac{5}{128} 1^{-7/2} 1^4$

$= 1 + \frac{1}{2} - \frac{1}{8} + \frac{1}{16} - \frac{5}{128} = \frac{179}{128}$

$179 + 128 = 307$ **D**

11) When $p(x)$ is divided by $x - a$, the remainder is $p(a)$.

$p(a) = 2^7 + 3\cdot 2^5 - 5\cdot 2^4 - 7\cdot 2^3 + 2\cdot 2^2 - 64$

$= 32$ $\quad$ – or –

2	1	0	3	-5	-7	2	0	-64
		2	4	14	18	22	48	96
	1	2	7	9	11	24	48	32

C

12) $\begin{vmatrix} \sin 7\theta & \cos 7\theta \\ \sin 2\theta & \cos 2\theta \end{vmatrix} = \sin 7\theta \cos 2\theta - \sin 2\theta \cos 7$

$= \sin 5\theta$ **A**

$\tan 2\theta = \frac{B}{A-C}$

$= \frac{50\sqrt{3}}{11 - {}^{-}39}$

$= \sqrt{3}$

$2\theta = \pi/3 \qquad \theta = \pi/6$ [A]

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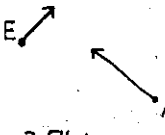
n steps taken. 5 to the left and own, 5 to the right direction.
$\binom{10}{5}$ ways to pick 5 steps in a tain direction and $\therefore \binom{10}{5} = 252$ ths. [D]



Let E be (0, 0)
M is (31.07, -10.93)
$P_{m_x} = 31.07 - 12t$
$P_{m_y} = -10.93 + 9t$

: 3.54 t
3.54 t

1. For some t, is $P_{M_x} = P_{E_x}$ ∧ $P_{m_y} = P_{E_y}$?

$.07\not{6} - 12t = 3.54t \qquad -10.93 + 9t = 3.54t$
$t = 2.00 \qquad t = 2.00$
they collide at $t = 2.00$.

$V_M = \sqrt{12^2 + 5^2} = 13 \qquad V_E = 3.54\sqrt{2} = 5.00$
$V_0 - V_M = \frac{13}{5} V_E$, not $4V_E$.

$: \sqrt{(31.07 - 12t - 3.54t)^2 + (-10.93 + 9t - 3.54t)^2}$
$t = 1, \ d = 16.47$ Yes
nd III are true. [D]

$= 01 \quad 7^4 = ...01$
$: 07 \quad 7^5 = ...07 \qquad 1997 \equiv 1 \bmod 4$
$\cdot 49 \quad 7^6 = ...49 \qquad \therefore \ ...07$
$...43 \quad \vdots$ [A]

Jrite 1,000,000 in base 7 : 1133311
e of 7^7
x6 3 each of $7^5, 7^4, 7^3, 7^2$, one $7 and
$1 prize.
n of the digits of the base 7 esentation.) = 16 [A]

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18) Horizontal, opens to right.
y coordinate of vertex is $\frac{7+{}^{-}1}{2} = 3$

$4a(x-h) = (y-3)^2$
$4a(3-h) = (5-3)^2 \qquad a = \frac{1}{3-h}$
$4a(6-h) = (7-3)^2 \qquad a = \frac{4}{6-h}$
$\frac{1}{3-h} = \frac{4}{6-h} \qquad 6-h = 12-4h \qquad 3h = 6 \qquad h = 2$
$a = \frac{1}{3-2} = 1 \qquad 4(x-2) = (y-3)^2$ [A]

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19) $\cos a \cos b = \frac{1}{2}(\cos(a+b) + \cos(a-b))$

$\cos 5\theta \cos 2\theta = \frac{1}{2}(\cos 7\theta + \cos 3\theta)$
$= \frac{1}{2}(1 + \frac{1}{2}) = \frac{3}{4}$ [D]

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20) $\frac{1}{r^2} + \frac{1}{s^2} + \frac{1}{t^2} = \left(\frac{1}{r} + \frac{1}{s} + \frac{1}{t}\right)^2 - 2\left(\frac{1}{rs} + \frac{1}{rt} + \frac{1}{st}\right)$
$= \left(\frac{rs + rt + st}{rst}\right)^2 - 2\left(\frac{r+s+t}{rst}\right)$
$= \left(\frac{6/8}{3\sqrt{3}/8}\right)^2 - 2\left(\frac{4\sqrt{3}/8}{3\sqrt{3}/8}\right) = {}^{-}4/3$ [A]

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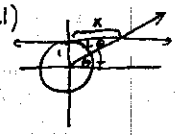
21)



∠s congruent ∵ alternate interior angles
$\tan\theta = \frac{1}{x} \qquad x = \cot\theta$
[B]

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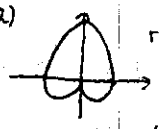
22)



$r = 2 + 2\sin\theta$,
CARDIOID [D]
(all curves of the form $r = a(1 \pm \sin\theta)$ or $r = a(1 \pm \cos\theta)$

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23) $A = rs = \sqrt{s(s-a)(s-b)(s-c)} \qquad s = \frac{5+7+8}{2}$
$10r = \sqrt{10(5)(3)(2)} \qquad = 10$
$r = \sqrt{3}$ [C]

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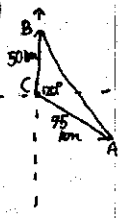
24)



$d^2 = 50^2 + 75^2 - 2 \cdot 50 \cdot 75 \cdot \cos 120°$
$d = \sqrt{11875}$
$= 25\sqrt{19}$
[B]

$\cos 3\theta + i \sin 3\theta = \cos^3\theta + 3\cos^2\theta \sin\theta i - 3\cos\theta \sin^2\theta - i \sin^3\theta$

COMPARE COEFFICIENTS OF i

$\sin 3\theta = 3\cos^2\theta \sin\theta - \sin^3\theta$ **C**

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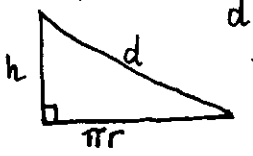
26) 

"unroll" the surface of the cylinder to get a right triangle



$d = \sqrt{15^2 + 8^2}$

$= 17$ **E**

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27) $\frac{4\sin^2\theta - \sin^2 2\theta}{\sin^2 2\theta} = \sec^2\theta - 1 = \tan^2\theta$

$\frac{\sin^2\theta - 1}{\sin^2\theta} = 1 - \csc^2\theta = -\cot^2\theta \neq \tan^2\theta$

$\frac{1 - \cos^2\theta}{\cos^2\theta} = \sec^2\theta - 1 = \tan^2\theta$

$\frac{\tan^2\theta - \sin^2\theta}{\sin^2\theta} = \sec^2\theta - 1 = \tan^2\theta$

Exactly three equal $\tan^2\theta$. **C**

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28) Pressure decreases with increasing volume. This eliminates choice D. The slope of the graph is not constant.

$m = \frac{3.0 - 2.1}{6.0 - 9.0} = -.3$ $\quad m = \frac{2.1 - 1.6}{9.0 - 12} = -.167$

$m = \frac{1.6 - 1.2}{12 - 16} = -.1$ $\quad m = \frac{1.2 - .97}{16 - 20} = -.058$

This means the relationship is not linear, eliminating choices A and B.

Plugging into C, we get (20, .96), (16, 1.2), (12, 1.5), (9, 2.0), and (6.0, 3.0).

C is clearly the best choice. **C**

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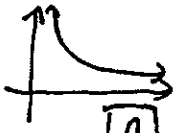
29) Only one graph is decreasing. Choice A is correct.



A

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30) Do a power regression on the data. The r^2 value given by the calculator is .9993

B

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1. Input the data:
   Press [STAT]
   Chose Edit... from the menu.
   Arrow up and [CLEAR] to clear L_1
   then →, ↑, [CLEAR], [enter] to cle
   arrow left. (↵ means [ENTER])
   20↵, 16↵, 12↵, 9↵, 6↵
   arrow right
   .97↵, 1.2↵, 1.6↵, 2.1↵, 3

2. Calculate a regression equation.
   [STAT], CALC, PwrReg, ↵, L_1, [9]
   This gives $y = 16x^{-.94}$ *

3. Graph the equation. Press [Y=].
   Clear or turn off all functions.
   Press [VARS], 5, →, →, 7 (1 on the (EQ))
   Press [2nd], [STAT PLOT], ↵.
   Select On, ⋰, L_1, L_2, □ optior
   Press [ZOOM]. Choose ZoomStat.

* 29

4. Square the stat variable r:
   [VARS], 5, →, →, r (6 on 82)
   (EQ)
   then [x^2], ↵

.999327371 * 30