CALCULUS INDIVIDUAL

VERO BEACH FAMAT INVITATIONAL **MARCH 25, 1995**

IF NONE OF THE GIVEN CHOICES IS CORRECT, CHOOSE E

(1) If the period of the function $g(x) = k\cos(2x/k)$ is 3, find its amplitude.

A) $3/\pi$ B) $2/\pi$

C) $\pi/3$ D) $1/3\pi$

(2) $x/\sqrt{x^2 + 9}$ is the derivative of

A) $\sqrt{x^2 + 9}/2$ B) $x^2/2\sqrt{x^2 + 9}$

C) $\sqrt{x^2 + 9}$ D) $2\sqrt{x^2 + 9}$

(3) Find y' when $x = 1$ and $y = \pi/2$, given $2xy + \pi \sin y = 2\pi$.

A) $\pi/2$ B) $-\pi/2$

C) $-\pi$ D) $-\pi/4$

(4) The function $f(x) = \dfrac{x^2 - 3x + 2}{x^3 - x}$ has the asymptotes

A) $x=0$, $x=1$, $x=-1$, $y=0$ B) $x=0$, $x=2$, $x=-1$

C) $x=0$, $x=1$, and $y=0$ only D) $x=0$, $x=-1$, and $y=0$ only

(5) Find the area enclosed by the curves $y = \sqrt{x}$ and $y = -x + 6$.

A) 44/5 B) 7.3

C) 23/3 D) 22/3

(6) If c is the number defined by Rolle's theorem, then, for $f(x) = 2x^3 - 6x^2$ on the interval $0 \leq x \leq 3$, c is

A) 2 B) 1/2

C) $\sqrt{2}$ D) $\sqrt{3}$

(7) If $f(x) = \tan^{-1}\left(\frac{1 - x}{1 + x}\right)$, then $f'(x)$ equals

A) $\frac{1}{1 + x^2}$ B) $-\frac{1}{1 - x^2}$

C) $-\frac{1}{1 + x^2}$ D) $-\frac{2}{1 + x^2}$

(8) The derivative of a function f is given for all x by $f'(x) = x^2(x + 1)^3(x - 4)^2$. The set of x for which f is a relative maximum is

A) $\{0,-1,4\}$ B) $\{-1\}$

C) $\{0,4\}$ D) $\{1\}$

(9) Woodstock is on top of a lob lolly pine 80 feet above a lake watching Snoopy in a motorboat moving directly away from the base of the tree at a rate of 25 feet per second. How fast is the angle of depression of Woodstock's line of sight changing when the boat is 150 feet from the base of the tree?

A) -0.08 rad/sec B) -0.03 rad/sec

C) -0.05 rad/sec D) -0.07 rad/sec

(10) Find $\frac{d^2y}{dx^2}$ if $y = \sqrt{x^2 + 16}$.

A) $4(3x^2 + 16)$ B) $\frac{16}{\sqrt{x^2 + 16}}$

C) $-\frac{1}{4(x^2 + 16)^{1.5}}$ D) $\frac{16}{(x^2 + 16)^{1.5}}$

(11) Find $G'(\pi/4)$ given $G(x) = \int_{\tan x}^{e} (u + 1)\, du$

A) -4 B) -2

C) 1/2 D) 4

(12) Determine the slope of the normal to the curve $f(x) = \dfrac{e^{\cos x}}{\sin x}$ at $(\pi/2, 1)$.

A) -1 B) e

C) 2 D) 1

(13) Suppose that a population of fruit flies increases exponentially. If there are 1000 flies initially and the population doubles every 4 days, how many days will it take for the population to grow to 10000 flies?

A) 4ln 10/ln 2 B) 12.9

C) 10.9 D) 8ln 5

(14) Find dy/dx if $y = x^{\ln x}$.

A) 2/x B) (2 ln x)/x

C) 2(ln x)y/x D) 2y/x

(15) The volume generated by rotating the region bounded by $y = 1/\sqrt{x}$, $x = 1$, $x = 4$, and $y = 0$ about the line $x = 0$ is

A) $20\pi/3$ B) $28\pi/3$

C) $34\pi/3$ D) $38\pi/3$

(16) Find: $\lim_{x \to 0} \dfrac{\int_0^x (\sin t - t)\, dt}{x^4}$

A) 0 B) -1/12

C) 1/24 D) -1/24

(17) Which of the following is the correct integral for finding the surface area of $f(x) = x^2 + 2x$ from $x = 1$ to $x = 3$ about the line $y = -2$?

A) $2\pi \int_1^3 (x^2+2x+2) \sqrt{4x^2+8x+6}\, dx$ B) $\pi \int_1^3 (x^2+2x+2)^2\, dx$

C) $2\pi \int_1^3 (x^2+2x+2) \sqrt{4x^2+8x+5}\, dx$ D) $\pi \int_1^3 x(x^2+2x+2)^2\, dx$

(18) Which of the following is neither even nor odd?

A) $y = \cosh x$ B) $y = 3x^7 - 2\csc x + e^{\sin x}$

C) $y = e^{-2xx} - 4$ D) $y = (\ln x)^2 - 2$

(19) Find the arclength of $f(x) = \dfrac{2(x^2 + 1)^{3/2}}{3}$ from $x = 1$ to $x = 4$.

A) 45 B) 35

C) 42 D) 39

(20) Evaluate $\int x \ln x\, dx$.

A) $(x^2 \ln x)/4 - x^2/4 + C$ B) $(x^2 \ln x)/2 - x^2/2 + C$

C) $(x^2 \ln x)/4 - x^2/2 + C$ D) $(x^2 \ln x)/2 - x^2/4 + C$

(21) The range of $g(x) = \ln(\cos x)$ is

A) $(-\infty, -1]$ B) $(0, 1]$

C) $[0, 1]$ D) $(-\infty, 0]$

(22) Differentiate $h(x) = \dfrac{e^{ax}(a^2x^2 - 2ax + 2)}{a^3}$, where a is a nonzero constant.

A) $h'(x) = 2xe^{ax}$ B) $h'(x) = 2x^2e^x$

C) $h'(x) = x^2e^{ax}$ D) $h'(x) = xe^{ax}$

(23) Evaluate $\lim\limits_{x\to\infty} \left(\dfrac{x-3}{x}\right)^x$.

A) e^3 B) 3^e

C) $-3e$ D) e^{-3}

(24) The graph of $F(x) = 1/x - 1/x^2$ is concave down on

A) $(-\infty,0)$ and $(2,\infty)$ B) $(-\infty,0)$ and $(0,3)$

C) $(-\infty,0)$ only D) $(-\infty,0)$ and $(0,\infty)$

(25) A 30-pound monkey hangs at the end of a 20-foot chain that weighs 1/2 pound per foot. How much work does it do in climbing the chain to the top? Assume that the end of the chain is attached to the monkey.

A) 650 ft-lb B) 700 ft-lb

C) 625 ft-lb D) 1300 ft-lb

(26) Let $f(x) = 2\tan x$, $-\pi/2 < x < \pi/2$ and find $g'(2)$ if $g(x) = f^{-1}(x)$.

A) 1 B) 1/4

C) 1/2 D) 1/8

(27) Find: $\lim\limits_{n\to\infty} \sum\limits_{k=0}^{n} \dfrac{\pi \sin(k\pi/n)}{n}$.

A) 1/2 B) 1

C) 4 D) 2

(28) Evaluate $\int_0^1 \int_2^y \int_1^2 2x^2y \, dx \, dy \, dz$

A) $(4y - 7)/2$ B) $(7y^2 - 28)/3$

C) $(7y^2 + 14)/2$ D) $(28y^2 - 14)/3$

(29) Find the dimensions of a rectangle of maximum area that can be inscribed in the ellipse $x^2/16 + y^2/9 = 1$.

A) $3\sqrt{2}$ by $4\sqrt{3}$ B) $4\sqrt{3}$ by $3\sqrt{3}$

C) $4\sqrt{2}$ by $3\sqrt{2}$ D) $2\sqrt{2}$ by $3\sqrt{2}$

(30) Evaluate $\int_0^4 \frac{\sqrt{t}}{t + 1} \, dt$

A) $2(1 + \tan^{-1}2)$ B) $2(2 - \tan^{-1}2)$

C) 1.78 D) 1.80