

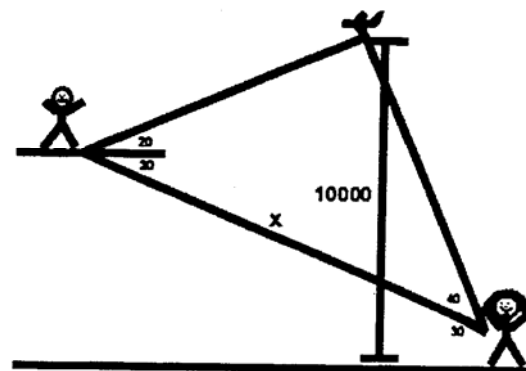
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- 1) **B** - The total area of the space and the border is equal to $(10 + 2 \cdot 2)(20 + 2 \cdot 2) = 336$. However, the area of the space itself is equal to $10 \cdot 20 = 200$. Therefore, the border alone has an area of $336 - 200 = 136$.
- 2) **C**-Eric, Stephanie, Russell, and Heather can sit in $4! = 24$ ways, assume the 4 person family is "one person" with the 6 other people. Thus, there are $(7-1)!$ ways to arrange around a circle. $24(6!) = 17,280$
- 3) **A** - $C = \pi d = 2\pi r = 2\pi \cdot \frac{400 \text{ meters}}{\pi} = 800$ meter circumference.

$$\frac{400 \text{ meters}}{\frac{2}{3} \text{ minutes}} \cdot \frac{1 \text{ revolution}}{800 \text{ meters}} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}} \cdot \frac{1}{60} \frac{\text{minutes}}{1 \text{ second}} = \frac{\pi \cdot 1 \text{ radians}}{40 \text{ seconds}}$$

$40 - 1 = 39$.



4) **C** - $\sin 70 = \frac{10000}{y}$ (y is the distance from the woman to the plane)

Angle measure from man in the triangle is $20 + 30 = 50^\circ$; so the angle measure from the plane in the triangle is $180 - 50 - 40 = 90^\circ$

$$\sin 50 = \frac{y}{x} \rightarrow x \approx 13,891.854.. \text{ (Tenth's digit is 8)}$$

- 5) **D** - Let $\frac{1}{2}x$ and x represent the number of weeks it takes for the father and son to build the car each by himself (respectively). Therefore, $\frac{6}{\frac{1}{2}x} + \frac{6}{x} = 1 \rightarrow \frac{12}{x} + \frac{6}{x} = 1 \rightarrow \frac{18}{x} = 1 \rightarrow x = 18$ weeks.

6) **D** - $\frac{\text{Volume}_{\text{fut}}}{\text{Volume}_{\text{Advisors}}} = \frac{\frac{1}{3}lwb}{\frac{1}{3}(.7l)(1.2w)(1.1b)} \cdot 100\% \approx 108.2\%$

- 7) **D** - Using the component method:

X coordinate	Y coordinate	Vector
$200 \cos 30^\circ$	$-200 \sin 30^\circ$	200 km @ 120°
$120 \cos 45^\circ$	$120 \sin 45^\circ$	120 km @ 45°
184.853°	-88.352°	258.502 @ 93.36°

- 8) **B** - The value of the Dell after $1\frac{3}{4}$ years is $\$3500 - \$1500 = \$2000$. $2000 = 3500e^{k(1.75)} \rightarrow \ln \frac{2000}{3500} = 1.75k \rightarrow k \approx -.3198$. $V_{1.75+1.5} = 3500e^{-.3198(1.75+1.5)} \approx \1240 . $2000 - x = 1240 \rightarrow x = \760 .

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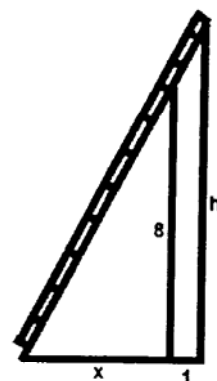
9) A – The ball keeps moving at 10 feet per second until the slower player reaches the net. The movement of this slower player can be represented by $d = rt \rightarrow \frac{1}{2}(60) = 2t \rightarrow t = 15$ seconds. If the ball moves for 15 seconds, it will move $10 \cdot 15 = 150$ feet.

10) B – Letting the clock represent a circle with the “12” at 0° , the sector in question is defined by the hour hand of 11:23, and the minute hand of 1:48. The hour hand is located at $\left(\frac{11}{12} \cdot 360 + \frac{1}{12} \cdot \frac{23}{60} \cdot 360\right) = 341.5^\circ$. The minute hand is located at $\left(\frac{1}{12} \cdot 360 + \frac{1}{12} \cdot \frac{48}{60} \cdot 360\right) = 54^\circ$. Together the hands form a $(360 - 341.5) + 54 = 72.5^\circ$ degree sector. The area of this sector is: $\frac{72.5}{360} \cdot \pi(8^2) = \frac{116\pi}{9} = \frac{580\pi}{45}$.

11) C – h can be found in terms of x by the following proportion: $\frac{8}{x} = \frac{h}{x+1} \rightarrow h = \frac{8x+8}{x}$.

An equation for the length of the ladder can then be written: $ladder = \sqrt{(x+1)^2 + \left(\frac{8x+8}{x}\right)^2}$.

The minimum value for the length of the ladder is then found to be ≈ 11 feet.



12) B – The probability that someone answers at least 10 of the 30 questions correctly is:

$$\sum_{10}^{30} \binom{30}{x} \left(\frac{1}{5}\right)^x \left(1 - \frac{1}{5}\right)^{30-x} \approx .0611. \text{ The expected number of people is then } .0611 \cdot 250 \approx 15 \text{ students.}$$

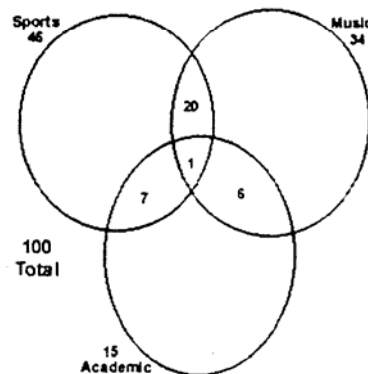
13) C – To find the velocity as the ball leaves the table: $v_f^2 = 2ad \rightarrow v_f = \sqrt{2(2.5)(5)} = 5$ feet per second. To find the time taken for the ball to reach the ground: $d = .5at^2 \rightarrow 3 = .5(32)t^2 \rightarrow t = \frac{\sqrt{3}}{4}$ seconds. Therefore,

$$d = rt = 5 \left(\frac{\sqrt{3}}{4}\right) = \frac{5\sqrt{3}}{4} \text{ feet.}$$

14) D – The volume of the torus is $V = \pi r^2 C = \pi \left(\frac{5}{2}\right)^2 (2\pi \cdot 8) = 100\pi^2$. The

volume of a cube is $V = s^3 = 100\pi^2$, so therefore a side of the cube is $\sqrt[3]{100\pi^2}$.

15) C – The number of students who played only sports is equal to $45 - 20 - 7 - 1 = 17$. The number of students who played only musical instruments is equal to $34 - 20 - 6 - 1 = 7$. The number of students who participated only on academic teams is equal to $15 - 7 - 6 - 1 = 1$. Therefore, the number who did not do any of the three activities is equal to $100 - 17 - 7 - 1 - 20 - 6 - 7 - 1 = 41$ students.



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16) C – Either Andrew and Sandip must miss it once then Viraj gets it, all three miss once while Andrew and Sandip miss it twice and Viraj gets it the second time, all three miss twice while Andrew and Sandip miss it thrice and Viraj gets it the third time, etc. The probability = $\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right) + \left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right) + \left(\frac{2}{3}\right)^8\left(\frac{1}{3}\right) + \dots$ (infinite sequence)

Therefore, the sum = $\frac{\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)}{1 - \left(\frac{2}{3}\right)^3} = \frac{4}{19}$.

17) C – Amplitude = leading coefficient = $\sqrt{\frac{2}{L}}$. Period = $\frac{2\pi}{\text{coefficient of } x} = \frac{2\pi}{\frac{n\pi}{L}} = \frac{2L}{n}$. The product of the

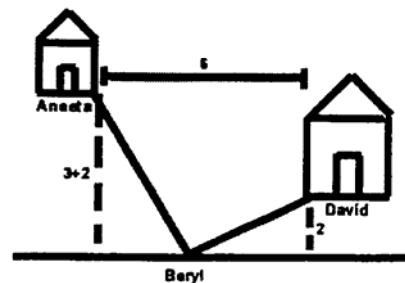
amplitude and the period is $\sqrt{\frac{2}{L}} \cdot \frac{2L}{n} = \sqrt{\frac{2L}{L^2}} \cdot \frac{2L}{n} = \frac{2L\sqrt{2L}}{nL} = \frac{2\sqrt{2L}}{n}$.

18) E – There are 36 possible paths from (0,0) to (4,4) that pass through (2,2).

19) A - $V = 288\pi = \frac{4}{3}\pi r^3 \rightarrow r = 6 \rightarrow S.A. = 4\pi r^2 = 144\pi$. The new surface area with lead coating must be three times this, or $412\pi = 4\pi r^2 \rightarrow r = 6\sqrt{3}$. Therefore the thickness (difference in radii) is $6\sqrt{3} - 6$ inches.

20) C – If $y = -2x^2 + 1$, let the sphere be represented by $x^2 + (-2x^2 + 1 - r) = r^2$. Solving for r yields $r = \frac{3x^2 - 4x^4 - 1}{4x^2 - 2}$. A relative extremum, the maximum value of r , occurs at $r = \frac{-1 + 2\sqrt{2}}{4}$ meters.

21) B – Let x represent the horizontal distance from Aneeta's house to Beryl, and let $6 - x$ represent the horizontal distance from Beryl to David's house. Using the Pythagorean Theorem twice, the total distance traveled can be represented by $d = \sqrt{x^2 + 5^2} + \sqrt{(6-x)^2 + 2^2}$. The minimum distance is therefore $\sqrt{85}$ miles.



22) D – If the span of the parabolic arch is 522 feet, the horizontal distance from the vertex to one of the bases is half this distance, or 261 feet. Let (70,560), (-261,0), and (261,0) represent points on the arch. Simultaneously solving the 3 equations

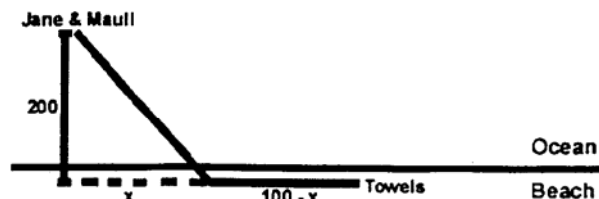
1) $560 = 70^2 a + 70b + c$, 2) $0 = (-261)^2 a - 261b + c$, 3) $0 = 261^2 a + 261b + c \rightarrow a \approx -.0089, b = 0, c \approx 603.4$ feet.

23) E – Total time = $\frac{\text{Distance in Water}}{\text{Rate in Water}} + \frac{\text{Distance on Land}}{\text{Rate on Land}}$.

Let the distance in water be found by the Pythagorean Theorem, so that the equation can be written: Total time =

$\frac{\sqrt{x^2 + 200^2}}{70} + \frac{100 - x}{110^2}$. The minimum time occurs when $x =$

100 meters.



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24) D – The spaceship touches the surface when $y = 0$. Setting $-0.01t^3 + .9t^2 - 25t + 250 = 0 \rightarrow t = 50$. As ϕ gets smaller and smaller, $v(50)$ gets closer to exactly -10 kilometers per second. Therefore the hundredth's digit is 0.

25) A – Let the total distance be represented by d . The total time would then be $\frac{d}{300} + \frac{d}{350} + \frac{d}{400}$. Therefore, since the total distance is $3d$, the average rate for the whole trip would be $\frac{3d}{\frac{d}{300} + \frac{d}{350} + \frac{d}{400}} \approx 345.2$ mph.

26. B Let a side of a square be x ; then the circumference of the circle is $1000 - 4x$, and the radius of the circle is $\frac{1000 - 4x}{2\pi}$. So the total area is given by: T.A. = $x^2 + \pi \left(\frac{1000 - 4x}{2\pi} \right)^2$.

You can find the minimum on the calculator, which is approx. 140. Multiplying this by $\sqrt{2}$, to get the diagonal, yields 198 (to the nearest yd).

27) B – Set 4 as an arbitrary value for the radius of the sphere. Let h be half the height of the inscribed cylinder. The radius of the cylinder is therefore represented by $\sqrt{4^2 - h^2}$, and the volume of the cylinder can be represented by $V = \pi(\sqrt{4^2 - h^2})^2 \cdot h$. The maximum of this function is therefore $\frac{128\pi\sqrt{3}}{9}$. So the ratio of the volumes is

$$\text{equal to } \frac{\frac{4}{3}\pi \cdot 4^3}{\frac{128\pi\sqrt{3}}{9}} = \frac{6}{\sqrt{3}} = 2\sqrt{3}, \text{ and } 3 - 2 = 1.$$

28) A – Either she has the makeup in a drawer on the left and the curling iron in a drawer on the right, or she has a curling iron on the left and her makeup on the right. The probability of this occurring equals $\frac{2}{4} \cdot \frac{1}{3} + \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{4}$.

29) A – Let x represent the number of additional pizzas ordered. Therefore the total amount spent can be represented by Price = x pizzas $\cdot \frac{\$13.24 - \$.49x}{\text{pizza}}$. The maximum integral value of this function occurs when $x = 14$.

If there are 14 additional pizzas ordered, there are $14 + 1 = 15$ total pizzas.

$$30) D - r^2 = \frac{16}{1 + 3\sin^2 \theta} \rightarrow r^2 + 3r^2 \sin^2 \theta = 16 \rightarrow r^2(\sin^2 \theta + \cos^2 \theta) + 3r^2 \sin^2 \theta = 16 \rightarrow$$

$$4r^2 \sin^2 \theta + r^2 \cos^2 \theta = 16 \rightarrow 4y^2 + x^2 = 16 \rightarrow a = 1, b = 0, c = 4, d = 0, e = 0, f = -16 \rightarrow acf + bde = -64 + 0 = -64.$$