MU ALPHA THETA - MAINE '95 CALCULUS APPLICATIONS TOPIC TEST

1. What are all values of x for which the function $f(x) = x^2 - 3x - 4$ is decreasing?

a. $(-\infty, 1.5)$ b. (-1.5, 1.5) c. (-1, 4) d. $(1.5, \infty)$ e. not given

2. The area of a region bounded by the parabola $8 + 2x - x^2$ and x-axis is

a. $41\frac{2}{3}$ b. 36 c. 20 d. $9\frac{1}{3}$ e. not given

3. Find an equation of the tangent line to the curve $x^3 + y^3 = 9$ at the point (1, 2).

a. $4x - y - 2 = 0$ b. $x + 4y - 9 = 0$ c. $x + 4y - 6 = 0$ d. $x - 4y + 7 = 0$
e. not given

4. The volume V, of unmelted ice remaining from a melting ice cube after t seconds is $V = 2000 - 40t + 0.2t^2$. How fast is the volume changing when t = 40 seconds?

a. -26 cu. in./ sec. b. -24 cu. in. / sec c. 24 cu. in. / sec d. 120 cu. in./ sec
e. not given

5. A particle moves along the x-axis. Its velocity is given by

$$V(t) = \begin{cases} t^2 \text{ for } 0 \le t \le 2 \\ t + 2 \text{ for } t \ge 2 \end{cases}$$

If it starts at the origin, its position after 4 seconds is x =

a. $37\frac{1}{3}$ b. $12\frac{2}{3}$ c. 10 d. $2\frac{2}{3}$ e. not given

6. Approximate the area bounded by the graphs of $y = x^2 + 1, x = 2$, the x-axis, and the y-axis using four rectangles below the graph.

a. $\frac{14}{3}$ b. $\frac{23}{4}$ c. $\frac{15}{4}$ d. $\frac{15}{2}$ e. not given

7. The curve $y = e^{-x}$ is concave upward

a. only on intervals that do not include zero. b. only for x in $(-\infty, 0)$.
c. only for x in $(0, \infty)$ d. every value of x e. not given

8. The area of the region between the curves y = 3x and $y = x^3 - 6x^2 + 11x$ is

a. 5 b. 6 c. 7 d. 8 e. not given

9. If x represents the number of units producted in a given period, the profit p in dollars of a company from the period is given by the equation: $p = (500x - x^2) - \left(\frac{1}{2}x^2 - 72x + 3000\right)$. Estimate the change in profit if the production is increased from 115 to 120 units.

a. \$1085 b. \$1135 c. \$1167 d. \$1242 e. not given

10. The line x = c, where $c < 0$, intersects $f(x) = 2x^3 + 3x^2 - 9$ at point P and $g(x) = 4x^2 + 4x + 5$ at point Q. If a line tangent to $f(x)$ at point P is parallel to a line tangent to g(x) at point Q, find the value of c.

a. $\frac{-3}{4}$ b. $\frac{-2}{3}$ c. $\frac{-1}{2}$ d. $\frac{-1}{3}$ e. not given

11. The velocity function for a particle moving left and right along the x-axis is given by $v(t) = 2\pi \cos \pi t$. For what values of t, $0 \le t \le 2$, is the particle not moving?

I. 0.5 II. 1 III. 1.5 IV. 2

a. I and II only b. II and III only c. I and III only d. II and IV only
e. not given

12. What is the total distance traveled to the left by the particle in problem #11?

a. 2 b. 4 c. 8 d. 16 e. not given

13. What is the equation of the tangent line to $y = x^3\sqrt{1+x^3}$ at the point (2, 24)?

a. $y = 52x - 128$ b. $y = 52x + 128$ c. $y = 52x + 80$ d. $y = 52x - 80$
e. not given

14. Find the area in the first quadrant bounded by $y = \sin 3x$ and the x-axis from $x = 0$ to the first x-intercept on the positive axis.

a. $\frac{1}{3}$ b. $\frac{2}{3}$ c. 1 d. 2 e. not given

15. A conical container has a height of 9 cm and a diameter of 6 cm as shown. It is leaking water at the rate of one cubic centimeter per minute. Find the rate at which the water level h is changing when h equals 3 cm.

a. $-\pi$ b. $\frac{-1}{\pi}$ c. $\frac{1}{\pi}$ d. π e. not given

16. Find the volume, in cubic units, of the solid formed by revolving the region bounded by $y = x^3$, x = 2, and y = 1 about the y-axis. All answers are in cubic units.

a. $\frac{93\pi}{5}$ b. $\frac{120\pi}{7}$ c. $\frac{47\pi}{5}$ d. $\frac{62\pi}{5}$ e. not given

17. The slope of a curve at each point (x, y) is given by $f'(x) = 2x - 1$. If the minimum value of f is $\frac{15}{4}$, find the equation of the curve.

a. $y = x^4 - x + 4$ b. $y = x^2 - x$ c. $y = x^2 - x + \frac{7}{2}$ d. $y = x^2 - x - 4$
e. not given

18. Find the average value of $f(x) = x\sqrt{2x+1}$ on [0, 4].

a. 18 b. $\frac{16}{3}$ c. 12 d. $\frac{298}{15}$ e. not given

19. If $f''(x) = 10$, $f'(0) = 2$ and $f(0) = 3$, find the coefficient of the linear term of $f(x)$.

a. 2 b. 3 c. 4 d. 5 e. not given

20. Find the area bounded by $f(x) = 4 - x^2$ and $g(x) = 2x + 1$.

a. 8 b. $5\frac{1}{3}$ c. $6\frac{2}{3}$ d. $10\frac{2}{3}$ e. not given

21. *If* $f(x) = x(\ln x)^2$ then which one is true?

a. $f(x)$ has a relative minimum at $x = \frac{1}{e^2}$. b. $f(x)$ has a relative minimum at $x = \frac{1}{e}$.
c. $f(x)$ has an inflection point at $x = \frac{1}{e}$. d. $f(x)$ is never concave up.
e. not given

22. If $f(x) = x^{\frac{5}{3}} - 5x^{\frac{2}{3}}$, which of the following statements is true?
I. $f(x)$ has a horizontal tangent at $x = 0$
II. $f(x)$ has a cusp at $x = 0$.
III. $f(x)$ has exactly one critical number.

a. II and III only b. I and II only c. II only d. III only e. not given

23. Solve the first-order linear differential equation $y' + 3xy = 2x$.

a. $y = \frac{x^2}{2} + \frac{2C}{3x^2}$ b. $y = \frac{2}{3}C$ c. $y = e^{\frac{-2x^2}{3}} + \frac{2}{3}$
d. $y = \frac{2}{3}e^{3x^2} + \frac{2}{3}$ e. not given

24. A metallic right circular cylinder is continuously heated. If the height is 4 times the radius, what is the rate, in in/sec, when the volume is increasing at the rate of 24π in/sec and the radius is 1.5 inches?

a. $\frac{4}{9}$ b. $\frac{8}{9}$ c. $\frac{9}{8}$ d. $\frac{9}{2}$ e. not given

25. Find the definite integral that represents the arc length of the curve $y = \frac{1}{x}$ over the interval [1, 3].

a. $\int_1^3 \sqrt{1+(\ln x)^2}\,dx$ b. $\int_1^3 \sqrt{1+\frac{1}{x^2}}\,dx$ c. $\int_1^3 \sqrt{1+\frac{1}{x^4}}\,dx$

d. $\int_1^3 \sqrt{\frac{1}{x}+\frac{1}{x^4}}\,dx$ e. not given

26. The base of a triangle is increasing at the rate of 2 ft/sec while the height is decreasing at 4 ft. sec. How is the area of the triangle changing when its base is 10 feet and its height is 6 feet?

a. A is increasing at a rate of 14 square feet per second.
b. A is decreasing at a rate of 14 square feet per second.
c. A is increasing at a rate of 22 square feet per second.
d. A remains constant.
e. not given

27. Suppose a ball is thrown directly upward with a speed of 96 ft./sec and moves according to the law $y = 96t - 16t^2$, where y is the height in feet above the starting point, and t is the time in seconds after it is thrown. What is the greatest height reached by the ball?

a. 96 ft. b. 80 ft c. 112 ft d. 144 ft. e. not given

28. A floodlight is on the ground 45 meters from a building. A thief 2 meters tall runs from the floodlight directly towards the building at 6 m/sec. How rapidly is the length of his shadow changing on the building when he is 15 meters from the building?

a. 0.6 b. 0.8 c. -0.6 d. -0.8 e. not given

29. A particle moves along the x-axis so that its position at time t is given by: $x(t) = (1 + t^2)e^{-t}$. For what values of t will the particle be at rest?

a. no values b. 0 c. 0.5 d. 1 e. not given

30. While ice skating, Mr. Randy Long, (just a beginner), who happens to be six feet tall, decides to take a rest and leans against a pole next to the rink. His feet start to slip out from under him at the rate of 1 fps. How fast is his head sliding down the pole when his feet are 5 feet away from the pole?

a. $\frac{5}{\sqrt{11}}$ b. $5\sqrt{11}$ c. 5 d. $\sqrt{11}$ e. not given