

Mu Alpha Theta National Convention: Seattle, 1997
Calculus School Bowl Test

- Kitty and Jenny are traveling, but have only one horse between them. If Kitty rides for a certain period of time, then gets off, ties up the horse, and walks on, allowing Jenny to reach the horse do the same, ad infinitum. What fraction of the time is the horse resting if they walk at 6 kmph and ride at 12 kmph?
- In an old mathematics text, you find a division problem in which you cannot discern many of the digits, but can tell where they were. What was the value of the divisor?

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- Three people are shown 3 red hats and 2 black hats that are to be placed upon them. They are seated one behind the other, so that person A can see persons B and C, person B can see person C, and person C can see no one, and then the hats are placed upon them, one per person. Person A is then asked whether he knows what color his hat is. If he does not, person B is then asked what color her hat is, and if she does not know, then person C will be asked whether she knows her hat's color. What is the probability that person C is asked her hat's color?

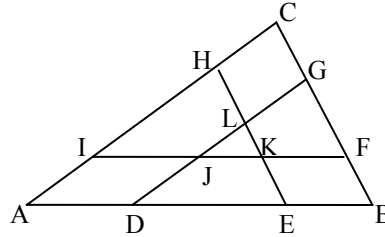
- Evaluate $\sqrt[3]{1 + \frac{\sqrt{5}}{2}} + \sqrt[3]{1 - \frac{\sqrt{5}}{2}}$.

- If A is the answer to problem 1, B is the answer to problem 2, C is the answer to problem 3, and D is the answer to problem 4, evaluate:

$$\frac{BD^3}{AC}$$

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6. Triangle ABC is cut by line segments IF, DG, and HE, which are parallel to sides AB, AC, and BC respectively. The areas of triangles BDG, AEH, and CFI are equal to one another, and to half the area of ABC. What is the ratio of the area of triangle JKL, formed by the intersections of IF, DG, and HE to that of triangle ABC?



7. What are the coordinates of the center of a sphere which passes through the points $(1, 2, 3)$, $(9, -2, 5)$, $(7, -4, -8)$, and $(-3, 4, -1)$?
8. What is the smallest integer greater than $(\sqrt{6} + \sqrt{5})^6$?
9. Let A be a number with an odd number of factors. A! ends in 898 zeros. How many factors does A have which are not divisible by 4?
10. If A is the answer to problem 6, B is the x-coordinate of the answer to problem 7, C is the answer to problem 8, and D is the answer to problem 9, evaluate:

$$\frac{C}{B} - AD$$

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11. Let $g(x) = x^5 + x^4 + x^3 + x^2 + x + 1$. What is the remainder when $g(x^{12})$ is divided by $g(x)$?

12. If θ is an acute angle and $\sin \frac{\theta}{2} = \sqrt{\frac{x+1}{2x}}$, then what is $\tan \theta$ in terms of x ?

13. What is the volume of the region bounded by the rotation of the graph of $y = x^2 - 225$ about the x-axis?

14. Evaluate: $\int \frac{x^3 + 2x^2 - 3x + 4}{x^2 - 5x + 6} dx$ for $x > 3$

15. If A is the answer to problem 11, B is the answer to problem 12, C is the coefficient of π in the answer to problem 13, and D is the coefficient of x in the answer to problem 14, evaluate:

$$\frac{C}{A} + DB$$

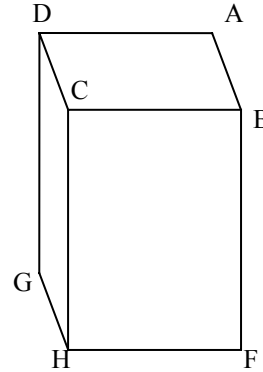
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16. In triangle ABC, a line is drawn through the center of the inscribed circle, parallel to BC, intersecting AB and AC at D and E respectively. What is the perimeter of triangle ADE, given $AB = 18$, $BC = 42$, and $AC = 30$?
17. What is the volume of a tetrahedron with vertices at the points $(3, 1, -2)$, $(4, 3, 4)$, $(2, -3, 5)$, and $(-1, 3, -5)$?
18. A student miscopies an equation of the form $x^2 - Ax + B = 0$, transposing the digits of B as well as the plus and minus signs. One of the roots he got was the same as one of the roots of the original equation. What was the root?
19. Triangle ABC is equilateral with side length 6. If circles of radius 4 are drawn centered at the vertices A, B, and C, what is the greatest distance between a point of intersection of two of the circles and a point of intersection of a different pair of circles?
20. If A is the denominator of the answer to problem 16, B is the answer to problem 17, C is the answer to problem 18, and D is the greatest integer less than the answer to problem 19, evaluate:

$ABCD$

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21. In the rectangular solid in the figure, the cosines of angles FHB and GHD are $\frac{1}{2}$ and $\frac{\sqrt{2}}{2}$ respectively. What is the cosine of angle BHD?



22. 5 dominoes, each 3 cm long, are piled upon one another. The dominoes will balance so long as the center of mass of all the dominoes above a given domino is somewhere above that domino (not to either side). What is the maximum attainable lateral distance, in cm, between the right edge of the top domino and the right edge of the bottom domino?
23. 4 men are shipwrecked on an island. They have gathered a bunch of coconuts, which they plan to divide the next day. During the night, each man, one after another, gets up, sneaks to the coconuts, and divides them into 4 shares. Discovering that there is 1 extra coconut, he gives it to a nearby monkey, then hides his share. The next morning, the remaining coconuts are divided among the castaways, leaving one coconut, which is left for the monkey. What is the minimum number of coconuts gathered?

24. If $A = \cos 36^\circ$ and $B = \cos 72^\circ$, determine $A-B$.

25. If A is the square of the answer to problem 21, B is the answer to problem 22, C is the answer to problem 23, and D is the answer to problem 24, evaluate:

$$\left(\frac{A+B}{D}\right)C$$

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26. In right triangle ABC, M is the midpoint of hypotenuse AB, N is the midpoint of AC, and BN is perpendicular to CM. If $BC = 8$, what is the length of BN ?

27. How many natural numbers are less than and relatively prime to 672?

28. A natural number, n , can be expressed as a three digit number in base x . When these three digits are reversed, and regarded as a base x number, it has twice the value of n . Find the value of n , expressed in base 10, such that the sum of the digits in base x is a minimum.

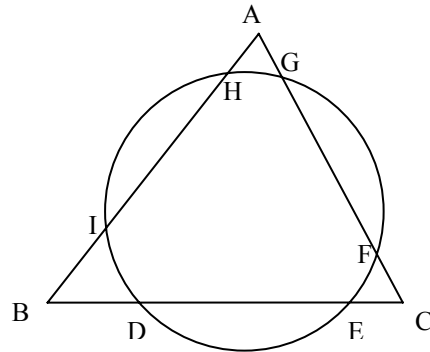
29. In the game “subtract a square”, a positive integer is written down and two players alternately subtract squares from it until a player is able to leave zero, in which case he is the winner. What is the optimal move for the first player, if the original number is 29?

30. If A is the answer to problem 26, B is the answer to problem 27, C is the answer to problem 28, and D is the answer to problem 29, evaluate:

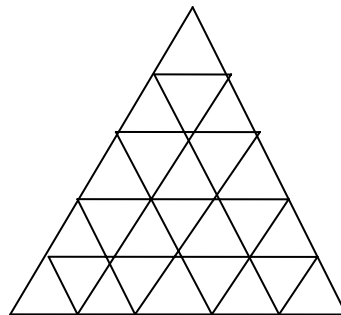
$$A + B + C + D$$

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31. In the figure, circle O intersects equilateral triangle ABC at D, E, F, G, H, and I. If $AG = 2$, $FG = 13$, $FC = 1$, and $HI = 7$, what is the length of DE ?



32. Determine N if the numbers N^3 and N^4 together contain each of the ten digits 0-9 exactly once.
33. Consider a room 10 meters wide, 20 meters long, and 12 meters high. If a spider is in the center of a 10 meter wide wall, one foot above the floor, and a fly is in the center of the other such wall, one foot from the ceiling, what is the shortest distance the spider can crawl to the fly?
34. In how many ways can a path be traced on the figure from A to B such that at all times it is getting farther from B and getting closer to A? Assume the smallest divisions all form congruent equilateral triangles whose areas are each $\frac{1}{25}$ of the area of the whole figure.



35. If A is the greatest integer less than the answer to problem 31, B is the answer to problem 32, C is the greatest integer less than the answer to problem 33, and D is the answer to problem 34, evaluate:

$$\frac{BCD}{A}$$

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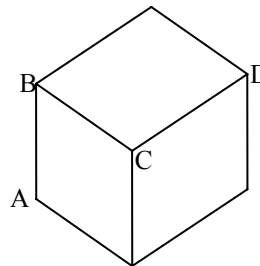
36. 7 towns, A-G are integer numbers of kilometers from one another along a straight highway, in the order A-G. If the towns are spaced such that the distance between any two of them is distinct, what is the minimum possible separation between A and G, in kilometers?
37. Two men sit down to play cards. The first wager is one cent, the second two cents, doubling with each bet. The gambling ends when one of the men, who brought only \$6.01 is cleaned out. If this is accomplished in the minimum number of bets, what was the largest number of cents in the loser's possession that night?
38. A point is chosen within a circle, and the chord of which that point is the midpoint is constructed. What is the probability that the length of the chord is greater than the length of a side of an inscribed equilateral triangle of the circle?
39. What is the sum of the cubes of the roots of $2x^3 + 3x^2 + 4x + 5$?
40. If A is the answer to problem 36, B is the answer to problem 37, C is the answer to problem 38, and D is the answer to problem 39, evaluate:

$$AB(2C + D)$$

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41. A point is chosen at random on a stick, and the stick is broken at that point. One of the two pieces is then chosen, and a point picked randomly on that piece, at which that piece is then broken. What is the probability that the three resulting pieces form a triangle?
42. An engineer must test each of three spacesuits for an hour at each of two different low pressure settings, and has two vacuum chambers available to him. If it takes 10 minutes to load a suit into a chamber, set the pressure, and start the test, 4 minutes to change the pressure of a chamber, and 10 minutes to unload a suit from a chamber, what is the minimum number of minutes in which he can complete the tests?
43. If $y = x^{\ln(x^x)}$, determine $y'(e^2)$

44. In the cube of the figure, let M, N, and O be the midpoints of AB, BC, and CD, respectively. What is the measure of angle MNO, in degrees?



45. If A is the answer to problem 41, B is the answer to problem 42, C is the answer to problem 43, and D is the answer to problem 44, evaluate:

$$A + B + C + D$$

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46. Given a regular dodecahedron which has been painted blue, how many distinguishable patterns can I produce by painting between one and three (inclusive) sides red?

47. A sphere of radius 6 is cut by a plane, such that their intersection is a circle with area 9π . What is the volume of the smaller of the two regions into which the plane divides the sphere?

48. Evaluate: $\int_0^{\sqrt{2\pi}} \int_{\frac{\sqrt{2\pi}}{2}}^{\sqrt{\pi}} \cos y^2 \, dy \, dx + \int_{\frac{\sqrt{2\pi}}{6}}^{\frac{3}{6}} \int_{3x}^{\sqrt{\pi}} \cos y^2 \, dy \, dx$

49. What is the probability that two points on the surface of a sphere subtend a central angle of less than 30° ?

50. If A is the answer to problem 46, B is the coefficient of π in the answer to problem 47, C is the answer to problem 48, and D is the answer to problem 49, evaluate:

$$B - D - \frac{A}{C}$$