

Mu Alpha Theta National Convention 2004
Mu School Bowl

1. Find $\lim_{h \rightarrow 0} \frac{(2+h)^5 - 32}{h}$

2. In a certain sequence of numbers, the first number is 1, and for all $n \geq 2$, the product of the first n numbers is n^2 . What is the sum of the fourth and fifth terms in this sequence? Express your final answer as a fraction in lowest terms.

3. If $xy + y^2 = 2$, find $\frac{d^2y}{dx^2}$ at (1,1).

4. Let $g(x) = \frac{x-4}{x^2-a^2}$. The graph of $g(x)$ has a removable discontinuity at $x = a$, a vertical asymptote at $x = b$, and a horizontal asymptote at $y = c$. Find $a + b + c$.

5. A committee consisting of 3 people is to be chosen at random from a department containing 6 women and 3 men. The probability that the committee consists of exactly 2 women and 1 man can be written as $\frac{A}{B}$, where A and B are relatively prime integers.

Find $A + B$.

6. Josie is running a carnival game. She rolls a pair of fair dice. She will pay n dollars if the sum of the two die is the product of n primes (not necessarily distinct). She will collect 3 dollars if the sum of the two die is a prime number. Find the mathematical expectation from Josie's viewpoint in dollars.

7. Find $\int (x^2 \ln x) dx$.

8. Find the volume of the solid that results when the region between the graph $y = \cos^2 x$ and $y = \sin^2 x$ from $x = 0$ to $x = \frac{\pi}{4}$ is revolved around the y-axis.

9. Find the slope of the line normal to $1 + 16x^2 y = \tan(x - 2y)$ at $(\frac{\pi}{4}, 0)$.

10. Find $\int \frac{dx}{\sqrt{4-9x^2}}$.

11. The region bounded by $y = x - x^2$ and $y = 0$ is revolved about the x-axis. Find the resultant volume.

12. Find the average value of $f(x) = x \ln x$ on the interval $[1, e]$.
13. Let $a = \cos t$ be the acceleration of a particle moving on a straight line such that the particle is at rest at $t = 0$. How far does the particle travel from $t = 0$ to $t = 3$?
14. Find the absolute minimum of $h(x) = axe^{ax}$ where $a \neq 0$.
15. Given the equation $x^2 - 2xy + 4y^2 = 64$, find $\frac{d^2y}{dx^2}$ at $(0, 4)$.
16. A hyperbola has asymptotes $2x - 5y = -23$ and $y = -\frac{2}{5}x + \frac{7}{5}$. One of its vertices is at $(-4, 1)$. The equation of this hyperbola can be written in the form $\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$. Find the sum of $a + b + h + k$.
17. The width of a rectangle is increasing at a rate of 2 cm and its length is increasing at a rate of 5 cm/sec. Let C cm²/sec be the rate of change in the area of the rectangle when the width is 4 and the length is 3. Let B cm/sec be the rate of the change in the length of the diagonal when the width is 4 and the length is 3. Find $C+B$.
18. Let $f(x) = e^x + x$. Find the value of $\frac{df^{-1}}{dx}$ at the point $(f(\ln 2), \ln 2)$.
19. Evaluate $\int_0^1 \frac{x^3}{\sqrt{x^4 - 9}} dx$.
20. If $(y \ln y)dx + (1 + x^2)dy = 0$ and $y(1) = e$, find $y(0)$.