Combinatorics-Open
2000 National Mu Alpha Theta Convention

1) How many multiples of 3 are between 38 and 700?

A) 221 B) 220 C) 219 D) 218 E) NOTA

2) Evaluate: $\binom{4}{0}+\binom{4}{1}+\binom{4}{2}+\binom{4}{3}+\binom{4}{4}$.

A) 12 B) 8 C) 16 D) 24 E) NOTA

3) How many different integers can be expressed as the sum of two distinct positive integers less than 20?

A) 35 B) 36 C) 37 D) 38 E) NOTA

4) How many factorials of positive integers are not divisible by 27?

A) 27 B) 8 C) 26 D) 9 E) NOTA

5) What value of n maximizes $\binom{12}{n}$?

A) 3 B) 7 C) 9 D) 12 E) NOTA

6) Central High School brings 5 ninth graders, 6 tenth graders, 7 eleventh graders, and 8 twelfth graders to the Mu Alpha Theta convention. If a four-person school team is made up of one person from each grade, how many different teams can Central make?

A) 26 B) 1680 C) 70 D) 420 E) NOTA

7) In a round-robin tournament, every player plays every other player once. If matches each take one hour, are allowed to proceed simultaneously, but each player can only play one match at a time, how long will a 9 person round-robin tournament take?

A) 8 hours B) 9 hours C) 16 hours D) 10 hours
E) NOTA

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8) How many lattice points inside (not on) the square which has (2,4) and (24,26) as vertices at the endpoints of one diagonal have two prime numbers as coordinates?

A) 72 B) 64 C) 81 D) 16 E) NOTA

9) How many ways can six books of different lengths be placed in a row such that no book is to the right (not necessarily directly) of a shorter book?

A) 0 B) 1 C) 120 D) 720 E) NOTA

10) How many factorials of positive integers have less than 10 terminating zeroes?

A) 51 B) 50 C) 45 D) 44 E) NOTA

11) I can climb stairs one step, two steps, or three steps at a time. In how many ways can I climb a flight of six stairs (3 steps, then 2, then 1 is considered different than 1 step, then 2, then 3)?

A) 7 B) 5 C) 24 D) 22 E) NOTA

12) Evaluate:

$$\sum_{m=1}^{10}\sum_{n=0}^{m}\binom{m}{n}$$

A) 2047 B) 511 C) 4095 D) 1023 E) NOTA

13) What is the probability that randomly formed two by two matrix in which each entry is a positive integer less than 101 has an even determinant?

A) ½ B) 3/8 C) 5/8 D) ¼ E) NOTA

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14) We say the set of non-positive integers has the same cardinality as the set of positive integers because we can create a one-to-one mapping of the first set to the second. Namely, map each member, n, in the first set to $-n+1$ in the second. Which of the following statements are true?

I) The set of integers has the same cardinality as the set of positive integers.
II) The set of all real numbers has the same cardinality as the set of real numbers between 0 and 1.
III) The set of integers has the same cardinality as the set of rational numbers.

A) none of these B) II + III C) I + II D) I only E) NOTA

15) There are six people in a room. Which of the following is closest to the probability that at least two of the six share a birthday?

A) 0.05 B) 0.1 C) 0.25 D) 0.5 E) NOTA

16) In how many ways can a ten-team round-robin tournament (each team places each other team once) proceed such that the first place team goes undefeated?

A) $5(2^{37})$ B) $7(2^{37})$ C) 2^{36} D) $3(2^{36})$ E) NOTA

17) I place a dime heads up on a table. I then flip a penny 9 times. Each time the penny comes up heads, I turn the dime over. What is the probability the dime is heads up when I am finished?

A) ½ B) 255/512 C) 257/512 D) 303/512 E) NOTA

18) How many integers less than 10000 have exactly 15 divisors?

A) 21 B) 12 C) 15 D) 14 E) NOTA

19) An integer is chosen at random from among the set of integers which are not divisible by the eleventh power of any prime and which have only three prime divisors. What is the probability that it is a perfect square?

A) 1/100 B) 0 C) $1/10^8$ D) 1/8 E) NOTA

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20) The Lakers and the Knicks play a game which the Lakers win 90-80. Given that all the scoring happened two points at a time, what is the probability that the game was tied at some score other than 0-0 at some point during the game?

A) 8/17 B) 8/9 C) 9/17 D) 16/17 E) NOTA

21) What is the probability that a lottery drawing which consists of a set of six numbers chosen from 1 to 40 does not contain a pair of consecutive numbers?

A) 1496/3367 B) 4488/9139 C) 13464/24605 D) 11594/27417
E) NOTA

22) How many different squares can be formed by connecting four vertices chosen from among lattice points (a,b) with $0 \le a, b \le 4$?

A) 16 B) 74 C) 30 D) 50 E) NOTA

23) 120 points are placed on the circumference of a circle such that they are the vertices of a regular 120-gon. Each subset of three or more of these points defines a convex polygon. How many of these polygons are regular?

A) 180 B) 158 C) 177 D) 213 E) NOTA

24) How many paths are there from (0,0,0,0) to (3,3,3,3) if each step in a path is taken by incrementing exactly one of the four coordinates by 1 and no paths pass through either of the points (1,2,3,1) or (3,3,3,2)?

A) 269640 B) 257040 C) 264612 D) 264600
E) NOTA

25) A standard deck of 52 cards has 13 ranks in each of four different suits. In how many ways can a five card hand be formed that has three cards of one rank and two cards of another (order of the cards in the hand does not matter)?

A) 156 B) 3744 C) 1872 D) 22464 E) NOTA

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26) Let n be a positive integer larger than 12. Let A be the set of all pairs (a,b) of even integers such that $a+b=2n$ and B be the set of all pairs (a,b) of odd integers such that $a+b=2n$.

Let $X = \sum_{(a,b)\in A} \frac{1}{a!b!}$ and $Y = \sum_{(a,b)\in B} \frac{1}{a!b!}$. Which of the following is always true:

A) $X > Y$ B) $X < Y$ C) $X = Y$
D) Whether X or Y is larger depends on the value of n.
E) NOTA

27) There are 64 teams in the NCAA basketball tournament. Whenever two teams play, the loser is out of the tournament and the winner proceeds to the next round. The tournament is over when there's only one team left. After two rounds, 16 teams are left. What fraction of the games that will take place in the entire tournament have already been completed?

A) 49/64 B) 3/4 C) 2/3 D) 16/21 E) NOTA

28) How many distinct powers of a are in the expansion of $\left(a^7 + a^{11} + a^{14}\right)^{20}$?

A) 126 B) 125 C) 130 D) 131 E) NOTA

29) Seven cars are placed on a one-lane highway at one mile intervals. They each start driving, one 10 mph, another 20 mph, another 30 mph, and so on, with the fastest going 70 mph. Each car drives its designated speed unless it catches the car in front of it, at which point the fast car slows to the slower speed. Thus, in a short time a steady state is reached of several clumps. If the cars are put in the seven starting points at random, what is the expected number of clumps formed? (For example, if the cars were paced from back to front as 20,70,60,40,10,30,50 then three clumps would form, one each headed by the 10, 30, and 50 mph cars.)

A) 363/140 B) 12023/5040 C) 3191/1260 D) 1993/720 E) NOTA

30) I grab a six-sided die and roll it repeatedly. What is the probability that I will roll four 1's before rolling my first 6?

A) 1/5 B) 1/32 C) 1/16 D) 1/4 E) NOTA