

1. What is the smallest positive integer n such that $\sqrt{n} - \sqrt{n-1} < .01$?
- a) 2499 b) 2500 c) 2501 d) 10000 e) There is no such integer
2. The number of distinct pairs (x, y) of real numbers satisfying both of the following equations $x = x^2 + y^2$ is $y = 2xy$
- a) 0 b) 1 c) 2 d) 3 e) 4
3. If $x < 0$, then $|x - \sqrt{(x-1)^2}| = \underline{\quad? \quad}$.
- a) 1 b) $1 - 2x$ c) $-2x - 1$ d) $1 + 2x$ e) $2x - 1$
4. If a, b, c , and d are non-zero numbers such that c and d are solutions of $x^2 + ax + b = 0$ and a and b are solutions of $x^2 + cx + d = 0$, then $a + b + c + d = \underline{\quad? \quad}$.
- a) 0 b) -2 c) 2 d) 4 e) $\frac{-1 + \sqrt{5}}{2}$
5. If $3^{2x} + 9 = 10(3^x)$, then the value of $(x^2 + 1)$ is
- a) 1 only b) 5 only c) 1 or 5 d) 2 e) 10
6. If $|x - \log y| = x + \log y$, where x and $\log y$ are real, then
- a) $x = 0$ b) $y = 1$ c) $x = 0$ and $y = 1$ d) $x(y - 1) = 0$
e) none of the above
7. For all real x , the inequality $1 \leq |x - 2| \leq 7$ is equivalent to
- a) $x \leq 1$ or $x \geq 3$ b) $1 \leq x \leq 3$ c) $-5 \leq x \leq 9$
d) $-5 \leq x \leq 1$ or $3 \leq x \leq 9$ e) $-6 \leq x \leq 1$ or $3 \leq x \leq 10$
8. The value(s) of y for which the following pairs of equations, $x^2 + y^2 - 16 = 0$ and $x^2 - 3y + 12 = 0$ may have a real common solution are
- a) 4 only b) -7, 4 c) 0, 4 d) no y e) all y

9. If $a + bi$, ($b \neq 0$) are imaginary roots of the equation $x^3 + qx + r = 0$ where a , b , q and r are real numbers, then q in terms of a and b is
- a) $a^2 + b^2$ b) $2a^2 - b^2$ c) $b^2 - a^2$ d) $b^2 - 2a^2$ e) $b^2 - 3a^2$
10. For which real values of m are the simultaneous equations $y = mx + 3$ and $y = (2m - 1)x + 4$ satisfied by at least one pair of real numbers (x, y) ?
- a) all m b) all $m \neq 0$ c) all $m \neq \frac{1}{2}$ d) all $m \neq 1$ e) no values of m
11. Which of the following inequalities are satisfied for all real numbers a , b , c , x , y , and z which satisfy the conditions $x < a$, $y < b$, and $z < c$?
- I. $xy + yz + xz < ab + bc + ac$
 II. $x^2 + y^2 + z^2 < a^2 + b^2 + c^2$
 III. $xyz < abc$
- a) none are satisfied b) I only c) II only d) III only e) all are satisfied
12. If $a \neq b$, $a^3 - b^3 = 19x^3$ and $a - b = x$, which of the following conclusions is correct?
- a) $a = 3x$ b) $a = 3x$ or $a = -2x$ c) $a = -3x$ or $a = 2x$
 d) $a = 3x$ or $a = 2x$ e) $a = 2x$
13. Which positive numbers x satisfy the equation $(\log_3 x)(\log_x 5) = \log_3 5$?
- a) 3 and 5 only b) 3, 5 and 15 only c) only numbers of the form $5^n \cdot 3^m$,
 where n and m are positive integers
 d) all positive $x \neq 1$ e) none of these
14. If p and q are prime integers and $x^2 - px + q = 0$ has distinct positive integral roots, then which of the following statements is/are true?
- I. The difference of the roots is odd.
 II. At least one root is prime
 III. $p^2 - q$ is prime
 IV. $p + q$ is prime
- a) I only b) II only c) II and III only d) I, II and IV only e) all are true

22. If $x = t^{[1/(t-1)]}$ and $y = t^{[t/(t-1)]}$, $t > 0$, $t \neq 1$, a relation between x and y is:
- a) $y^x = x^{(1/y)}$ b) $y^{(1/x)} = x^y$ c) $y^x = x^y$ d) $x^x = y^y$
 e) none of the above

23. If n is a real number, then the simultaneous system to the right has no solution if and only if n equals
- $$\begin{cases} nx + y & = 1 \\ ny + z & = 1 \\ x & + nz = 1 \end{cases}$$
- a) -1 b) 0 c) 1 d) 0 or 1 e) $\frac{1}{2}$

24. Let $[t]$ denote the greatest integer $\leq t$ where $t \geq 0$ and $S = \{(x, y): (x - T)^2 + y^2 \leq T^2 \text{ where } T = t - [t]\}$. Then we have
- a) The point $(0, 0)$ does not belong to S for any t .
 b) The area of S is bounded by 0 and π .
 c) S is contained in the first quadrant for all $t \geq 5$.
 d) The center of S for any t is on the line $y = x$.
 e) None of the above statements is true.

25. In a certain scale of notation, $24^2 = 554$. The base of this scale is
- a) 6 b) 8 c) 12 d) 14 e) 16

TIEBREAKER

Solve the system

$$\begin{aligned} 4w + 3x + 5y - 7z &= -15 \\ 8w - 2x + 3y + 5z &= 40 \\ 2w + x - 4y - 4z &= -22 \\ 6w - 5x + 8y + z &= 34 \end{aligned}$$