FAMAT Regional Mathematics Competition - Calculus Team Round - February 8, 1997

1. Find the exact area bounded by the graphs $y = 10 - \frac{x^2}{2}$ and $y = \frac{x^2}{2} - 3x$.

2. How many times do the graphs $y = \sin(x)$ and $y = \ln(x^2)$ intersect?

3. Evaluate: $\int x^3\sqrt{x^{-2} + x}dx, x > 0$

4. Given the function $y = x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}$, if dy/dx is found for this equation and expressed in the form f(y)/x, where f(y) is only a function of y, then find f(y).

5. What is the volume of the solid formed by revolving the graph $r = 6\sin\theta$ about the line that passes through the points $(11, \pi)$ and $(5, 78\pi)$? (Coordinates are in polar form)

6. Find the exact value of $\int_0^{1/2} \arcsin(x)dx$.

7. Find $\frac{dy^2}{d^2x}$ for the equation $xy = x^2 + 2y$ at the point (3, 9).

8. Let $A = \int_0^{\pi} \sqrt{1 + \cos(x)}dx$.

   Let $B = f''(\sqrt{\pi/2})$, where $f(x) = \sin(\cos(x^2))$.

   Find $\int_B^A (x^3 + x)dx$.

9. Find tanC for a triangle ABC, with sides a, b, and c, in which $\frac{a^3 + b^3 + c^3}{a + b + c} = c^2$.

10. A solid is formed so that its intersection in three-dimensional space with the xy plane is the circle $x^2 + y^2 = 9$. If every cross-section of this solid, perpendicular to the x-axis is a regular hexagon with main diagonal in the xy plane, then what is the volume of this solid?

11. $\Gamma(n)$ is defined as $\Gamma(n) = \int_0^{\infty} x^{n-1}e^{-x}dx$. Find $\Gamma(6)$.

12. Find $f'(2)$ if $f(x) = \frac{x^2 \operatorname{arc\,csc}(x)}{x + 2}$.

13. A = the number of different team round scores that any one team may receive during a single team round at a sanctioned FAMAT regional mathematics competition.
    B = the number of ways that a math teacher can choose a team of four members from eight different students who are competing individually
    C = the number of different ways that eight different keys can be arranged on a key ring.
    Find BC/A

14. Over what intervals is the graph $y = x^3 - 2x^2 - 20x - 8$ both decreasing and concave up?

15. Find the derivative of y with respect to x^3 for the equation $xy + x^2y = 2y^2 - x$.