FEBRUARY REGIONAL 2005
GEOMETRY *INDIVIDUAL*

For all questions, choice **E** is "NOTA," meaning "None Of These Answers."

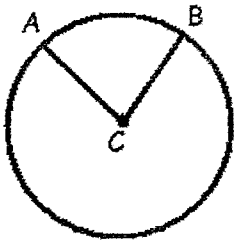
Diagrams are not necessarily drawn to scale!

1. A circle has a diameter of 6. What is the area of the circle?

A) 6π B) 9π C) 12π D) 36π E) NOTA

2. In a circle with center C, the measure of minor arc $\overset{\frown}{AB} = (2x+9)°$ and $m\angle ACB = (60-x)°$. What is the value of x?

A) 10.2 B) 17
C) 23 D) 51
E) NOTA



3. The same amount of fencing used to enclose a 20 by 12 rectangular area is used to enclose a square-shaped area. What is the ratio of the rectangle's area to the square's area?

A) 1:4 B) 15:16 C) 15:4 D) 15:1 E) NOTA

4. A quadrilateral has sides measuring $3b^2$, $4b$, $2b^2$, and b. Which of the following expresses the perimeter of the quadrilateral in terms of b?

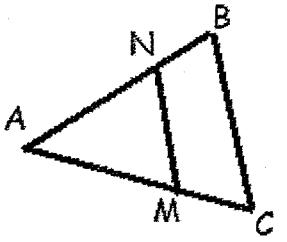
A) $5b^4+4b^2$ B) $6b^2+4b$ C) $9b^2$ D) $9b^6$ E) NOTA

5. A regular hexagon has an area of $18\sqrt{3}$. What is the measure of one of the hexagon's sides?

A) $\sqrt{3}$ B) $2\sqrt{3}$ C) $3\sqrt{3}$ D) 12 E) NOTA

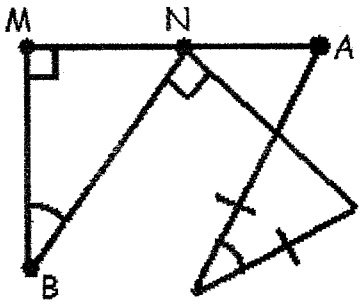
6. Points M and N lie on triangle ABC as shown in the diagram below. The quadrilateral $MNBC$ is a trapezoid. If $BC = 30$, $AM = 18$, and $MN = 12$, then what is the length of $\overline{AC}$?

A) 27 B) 36
C) 45 D) 63
E) NOTA



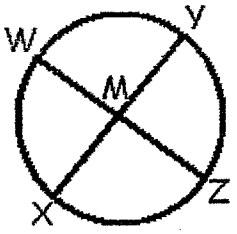
7. In the diagram below, $m\angle B = 40°$. M, N, and A are collinear, and sides and angles with the same markings are congruent. What is $m\angle A$?

A) 40° B) 50°
C) 60° D) 70°
E) NOTA



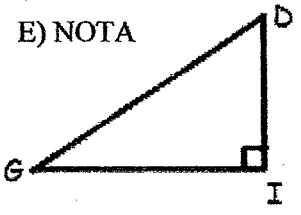
8. In the circle shown, chords $\overline{ZW}$ and $\overline{XY}$ intersect at M. $WM = 12$, $ZM = 15$, and $XM = 18$. What is the length of $\overline{XY}$?

A) 9 B) 10
C) 27 D) 28
E) NOTA



9. In right triangle DIG, $DI = 8$ and $GD = 17$. What is $\cos(D)$?

A) $\frac{8}{17}$ B) $\frac{8}{15}$ C) $\frac{15}{17}$ D) $\frac{15}{8}$ E) NOTA



10. $\Delta ABC \sim \Delta DEF$, and $m\angle B = m\angle E = 90°$. If $AB = 3$, $BC = 4$, and $EF = 12$, then what is the perimeter of the larger triangle?

A) 36 B) 48 C) 54 D) 56 E) NOTA

11. An angle measures $15°$ more than half its supplement. What is the measure of the angle?

A) $40°$ B) $55°$ C) $70°$ D) $76.\overline{6}°$ E) NOTA

12. Which of the following statements must be true?

I. The diagonals of a rectangle are perpendicular. II. A rhombus is equilateral. III. A pair of lines that are parallel are also skew.

A) I only B) II only C) III only
D) I and II only E) NOTA

13. A rhombus with a perimeter of 64 has an area of $28\sqrt{7}$. What is the product of the lengths of the diagonals of the rhombus?

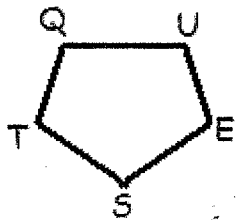
A) $32\sqrt{7}$ B) $56\sqrt{7}$ C) 128 D) $64\sqrt{7}$ E) NOTA

14. A right triangle with integral side lengths has one leg measuring 60. Which of the following could not be the length of its hypotenuse?

A) 68 B) 75 C) 87 D) 100 E) NOTA

15. In pentagon $QUEST$, $m\angle UQT = (3x)°$, $m\angle QTS = (2x+10)°$, $m\angle TSE = (x+40)°$, $m\angle SEU = (5x-35)°$, and $m\angle EUQ = (4x)°$. What is $m\angle EST$?

A) $63°$ B) $75°$
C) $87°$ D) $108°$
E) NOTA



16. A rectangle with dimensions 15 and 5 has a perimeter of $3x+2y$, and a semiperimeter of $2x+y$. What is the value of $(x+y)$?

A) 0 B) 10 C) 20 D) 60 E) NOTA

17. What is the next number in this sequence?

2, 3, 7, 16, 32, 57, ...

A) 79 B) 93 C) 100 D) 114 E) NOTA

18. $\overrightarrow{MJ}$ bisects $\angle EMT$, $\overrightarrow{MT}$ bisects $\angle JMP$, and $\overrightarrow{MA}$ bisects $\angle EMJ$. If $m\angle AMT = 30°$ and $m\angle EMP \leq 180°$, then what is $m\angle PMJ$?

A) $40°$ B) $60°$ C) $75°$ D) $120°$ E) NOTA

19. What is the distance between the three-dimensional coordinates (1, 1, 1) and (5, 3, 8)?

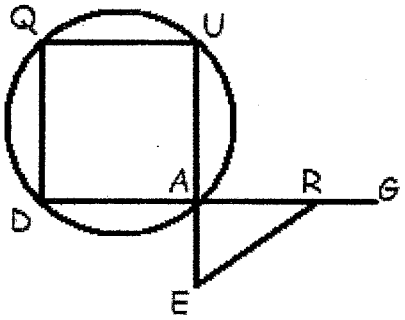
A) $2\sqrt{14}$ B) $\sqrt{69}$ C) $8\sqrt{2}$ D) 13 E) NOTA

20. The distance from the center of a circle to a chord in the circle is 8. The radius of the circle is 12. What is the length of the chord?

A) 8 B) $4\sqrt{5}$ C) $4\sqrt{10}$ D) $8\sqrt{5}$ E) NOTA

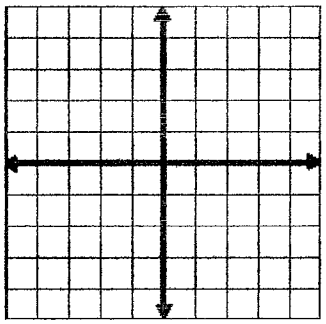
21. In the diagram below, quadrilateral $QUAD$ is inscribed in the circle shown, and $\overline{UE}$ intersects $\overline{DR}$ at A. If $m\angle E = 35°$, $m\angle D = 85°$, and $m\angle Q = 95°$, then what is $m\angle ERG$?

A) $120°$
B) $125°$
C) $130°$
D) $135°$
E) NOTA



22. A triangle exists whose vertices are the intersections of the lines $y = x + 1$, $y = -2x + 4$, and $x = 4$. What is the area of the triangle?

A) 13.5
B) 15
C) $4.5\sqrt{10}$
D) $12\sqrt{2}$
E) NOTA



23. A traditional analog clock reads 9:58. What is the measure of the acute angle formed by the hour hand and the minute hand?

A) 49° B) 51° C) 54° D) 56° E) NOTA

24. One base of a trapezoid measures 20. The other base measures 10. One side of the trapezoid measures 6, and the base angle adjacent to the same side measures 45°. What is the area of the trapezoid?

A) $45\sqrt{2}$ B) $90\sqrt{2}$ C) $135\sqrt{2}$ D) $180\sqrt{2}$ E) NOTA

25. A mysterious building is in the shape of a regular hexagon. Each side of the building measures 100 feet. A small dog (his name is "Ptolemy") is attached to a vertex of the building (on the outside) by a 50-foot leash. What is the greatest distance (in feet) that Ptolemy can walk with his leash fully taut without retracing his steps? Assume that the leash reaches exactly 50 feet fully taut, and disregard the size of the dog. Oh, and round to the nearest foot. (You can approximate pi to 3.14).

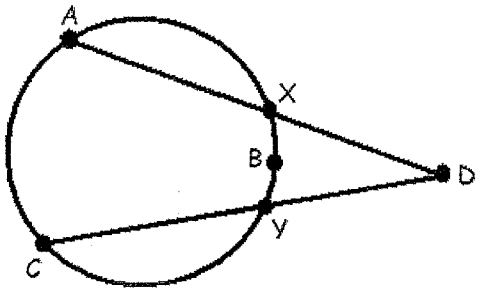
A) 115 B) 209 C) 226 D) 314 E) NOTA

26. A 30-60-90 triangle has an area of $18\sqrt{3}$. What is its perimeter? Round to the nearest tenth.

A) 15.8 B) 20.1 C) 22.4 D) 28.4 E) NOTA

27. $\overline{AD}$ intersects the circle shown below at X. $\overline{CD}$ intersects the circle at Y. $m\overset{\frown}{ACB} = 245°$, $m\overset{\frown}{BAC} = 260°$, $m\overset{\frown}{CBA} = 215°$, and $m\angle CDA = 35°$. What is $m\overset{\frown}{XY}$?

A) 30° B) 40° C) 55° D) 75° E) NOTA



28. How many distinct triangles exist which have side lengths measuring $\frac{7}{6}$, $\frac{2}{5}$, and $\frac{9}{11}$?

A) 0 B) 1 C) 2 D) 3 E) NOTA

29. A right pyramid with a square base has a height of 12 and a volume of 64. What is the length of the lateral edge of the pyramid?

A) $2\sqrt{37}$ B) $2\sqrt{38}$ C) $4\sqrt{10}$ D) $4\sqrt{11}$ E) NOTA

30. A student who is proving that two right triangles are congruent proves that they are congruent by inventing her own proof, "**LL**", with **L** denoting "leg." Although she had accurately stated that each pair of corresponding legs of the triangles are congruent in her proof, Mr. Rash marks her proof incorrect - because "**LL**" is not listed in his Magic Teacher's Edition as a valid proof. However, he decides to give her credit after the proud student calmly explains that "**LL**" is just a special case of what valid proof?

A) ASA B) HA C) SAS D) SSA E) NOTA