FAMAT REGIONAL
CALCULUS INDIVIDUAL
JANUARY 11, 1992

1. Give the domain of $\dfrac{\sqrt{x^2-9}\ \ln(25-x^2)}{x-4}$.

a) $[-5,-3]\cup[3,4)\cup(4,5]$ b) $(-5,-3]\cup[3,4)\cup(4,5)$
c) $(-5,-3]\cup[3,5)$ d) $[-5,-4)\cup(-4,-3]\cup[3,4)\cup(4,5)$ e) NOTA

2. $y = \ln(\arctan(x^2))$ Find y'.

a) $\dfrac{1}{(x^4+1)(\arctan(x^2))}$ b) $\dfrac{2x}{\arctan(x^2)}$ c) $\dfrac{1}{y(x^4+1)}$
d) $\dfrac{2x}{e^y(1+x^4)}$ e) NOTA

3. Find $\dfrac{dy}{dx}$ if $e^{xy} - x = 3$.

a) $\dfrac{1}{3+x}$ b) $\dfrac{1-e^{xy}y}{e^{xy}}$ c) $\dfrac{1-e^{xy}}{(3+x)y}$ d) $\dfrac{1-ye^{xy}}{e^{xy}x}$ e) NOTA

4. $y = 3^x + 2$ Write the equation of the tangent to the curve at $x = -1$.

a) $3\ln(3y) = x+1+7\ln(3)$ b) $\ln(3x)-3y = 3-\ln(3x)$
c) $\ln(3)x-3y = -\ln(3)-7$ d) $\ln(3x)+3y = -\ln(3)+5$ e) NOTA

5. Gas is escaping from a spherical balloon at a rate of $10ft^3/hr$. At what rate is the radius decreasing when the volume is $\dfrac{1000\pi}{3}ft^3$?

a) $\dfrac{1}{10\sqrt[3]{4}}$ b) $\dfrac{1}{40\sqrt[3]{2}}$ c) $\dfrac{\sqrt[3]{2}}{80\pi}$ d) $\dfrac{1}{10\pi\sqrt[3]{4}}$ e) NOTA

6. $\lim_{x\to 0^+}\left(\dfrac{\sqrt{x}}{\sqrt{4+\sqrt{x}}-2}\right) =$

a) 0 b) 1/4 c) 4 d) 8 e) NOTA

7. $f(x) = \sin(\sqrt{1+3x})$ Find $f'(0)$.

a) 0 b) 1/2 c) 3/2 d) $\dfrac{3}{2}\cos(1)$ e) NOTA

8. Find the absolute maximum value of f in the interval [-3,3] if $f(x) = 2x^3 - 6x - 1$.

a) -5 b) -1 c) 3 d) 37 e) NOTA

9. Find the slope of the normal to $y = \log_3 4x^2$ at $x = -2$.

a) $\frac{-1}{16\ln(2)}$ b) $\frac{-2}{\ln(3)}$ c) $-2\ln(3)$ d) $\ln(3)$ e) NOTA

10. A long rectangular sheet of metal, 12 inches wide, is to be made into a rain gutter by turning up two sides at right angles to the sheet. How many inches should be turned up on each side to give the gutter its greatest capacity?

a) 1 b) 1.5 c) 3 d) 3.5 e) NOTA

11. Find $f'(x)$ if $f(x) = 3x^{5/2} + 2x^{3/2} - 6x^{1/2}$.

a) $\frac{3(5x^2+4x-4)\sqrt{x}}{2}$ b) $\frac{30x^2+4x-4}{2\sqrt{x}}$ c) $\frac{3(5x^2+2x-2)}{2\sqrt{x}}$

d) $\frac{3\sqrt{x}(5x^2+x-2)}{2x}$ e) NOTA

12. A curve is given parametrically by $x = \ln(t)$ and $y = 2\ln(5t)$. The equation of the tangent to the curve at t=2 is

a) $2x-y = 2\ln 2 - 2\ln 10$ b) $(\ln 2)x-2\ln(5y) = 2\ln 5$
c) $5x-y = 5\ln 2 - 2\ln 10$ c) $5x+y = 10\ln 2 + 2\ln 10$ e) NOTA

13. $f(x) = x^3 + cx^2 + dx + e$. For what values of c and d will f have critical values at x=4 and x=-2 ?

a) c=-5,d=-8 b) c=-5,d=-12 c) c=-3,d=-24 d) $c=\frac{-3}{2}$,d=0 e)NOTA

14. In what interval is the value of f negative while the function is increasing? $f(x) = 2x\ln(x)$

a) $\left[0,\frac{1}{e}\right]$ b) (0,1) c) $\left[\frac{1}{e},e\right]$ d) (1,e) e) NOTA

15. Find $f''(1)$ if $f(x) = \frac{x+1}{x^2-3}$.

a) -4 b) -3 c) -5 d) -2 e) NOTA

16. Find y' if $y = \sin(x)^{\csc(x)}$.

a) $\csc(x)\cot(x)(1+\ln(\sin(x)))\sin(x)^{\csc(x)}$
b) $\cot(x)(1-\ln(\sin(x)))\sin(x)^{\csc(x)-1}$
c) $\csc(x)(\cot(x)-\ln(\sin(x)))\sin(x)^{\csc(x)}$
d) $\csc(x)(\csc(x)-\cot(x)\ln(\sin(x)))\sin(x)^{\csc(x)}$
e) NOTA

17. Let g be the geometric mean of x and x+n. Let a be the arithmetic mean of x and x+n. $\frac{dg}{dx}$ =

a) $\frac{a}{g}$ b) $\frac{g}{a}$ c) ag d) $\frac{a}{\sqrt{g}}$ e) NOTA

18. $\lim_{x\to-\infty}\left(\frac{5x-2}{\sqrt{3x^2+1}}\right)$ =

a) $\frac{-5}{3}$ b) $\frac{5}{\sqrt{3}}$ c) $\frac{5}{3}$ d) no limit e) NOTA

19. Find $y = f(x)$ if $f''(x) = x^{-3/2}$, $f'(1) = 2$, and $f(9) = -4$.

a) $y = \frac{-24}{\sqrt{x}} + 4$ b) $y = -4\sqrt{x} + 4x - 28$

c) $y = -2\sqrt{x} + 4x - 26$ d) $y = -4x^{1/2} + 4x - 24$ e) NOTA

20. Find the slope of the tangent line to $y = e^{3x}$ at $x = \ln(2)$.

a) $3e^2$ b) 18 c) 24 d) $3e^6$ e) NOTA

21. The yield V (in millions of cubic feet of wood per acre) for a forest at age t is given by $V = 9.2e^{(-28.1)/t}$ where t is measured in years. Find the limiting volume of wood per acre as t approaches infinity.

a) $\frac{9.2}{e}$ b) 9.2 c) 9.2e d) $\frac{\ln(9.2)}{-28.1}$ e) NOTA

22. For what values of k will the graph of $y = x^3 + kx^2 + x + 2$ have two horizontal tangent lines ?

a) $k=3$ b) $-3<k<3$ c) $k<-3$ or $k>3$ d) $|k| > \sqrt{3}$ e) NOTA

23. $\lim_{x\to5} f(x)$ is non-existent if $f(x)$ =

a) $\frac{x^2-25}{x-5}$ b) $\sin(x-5)$ c) $\frac{x}{x^2-5x}$ d) $\frac{x-5}{x}$ e) NOTA

24. In the interval $[0,\pi]$ what are the x coordinates of the points of inflection for $y = \sin^2(x) - \cos^2(x)$?

a) $0,\pi$ b) $\frac{\pi}{4},\frac{3\pi}{4}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{3},\frac{2\pi}{3}$ e) NOTA

25. If $f(x) = \arcsin(2x)$ and $g(x) = e^{x^2}$, then $f'(g(x)) =$

a) $\frac{2}{\sqrt{1-4e^{x^4}}}$ b) $\frac{2xe^{x^2}}{\sqrt{1-4e^{2x^2}}}$ c) $\frac{2xe^{x^2}}{\sqrt{1-4e^{x^4}}}$ d) $\frac{2}{\sqrt{1-4e^{2x^2}}}$ e) NOTA

26. Find the area bounded by $y^2 = 4x$ and $x^2 = 4y$.

a) $\frac{16}{3}$ b) $\frac{25}{3}$ c) 4 d) 3 e) NOTA

27. $\int_0^{\frac{\pi}{4}} \tan^2(x)\sec^2(x)\cot(x)dx =$

a) 0 b) $\frac{1}{2}$ c) $\frac{\pi^2}{8}$ d) $\frac{\pi^2}{32}$ e) NOTA

28. If $y = \sec(u)$, $u = 2w^2$, and $w = x^\pi$, then $\frac{dy}{dx} =$

a) $2\sec(2x^{2\pi})\tan(2x^{2\pi})$ b) $4\pi x^{2\pi-1}\sec(2x^{2\pi})\tan(2x^{2\pi})$

c) $4\pi^2 x^{\pi^2-1}\sec\left(2x^{\pi^2}\right)\tan\left(2x^{\pi^2}\right)$

d) $\pi x^{\pi-1}\sec(2x^\pi)\tan(2x^\pi)\sec(2x^{2\pi})$ e) NOTA

29. If $\int_0^x g(t)dt = \frac{1}{3}x^3 + 2$ and $f'(x) = g(x)$, then $\frac{d^2}{dx^2}\left[f(x^5)\right] =$

a) $70x^{13}$ b) $5x^{14}$ c) $5x^4$ d) $\frac{1}{3}x^5$ e) NOTA

30. Which of the following statements is false for $f(x) = \frac{4x^2-1}{x^2-9}$?

a) f is symmetric about the y axis.
b) The graph of f has 3 asymptotes.
c) f reaches a relative minimum at $x = 0$.
d) The roots of f are at $x = \frac{1}{2}$ and $x = -\frac{1}{2}$.
e) NOTA