ALPHA LOGS AND EXPONENTS
FAMAT STATE CONVENTION 2006

NOTA, "none of these answers," should be selected only if the correct answer is not given as one of the possible answers.

1. $f(x) = \log(x-1)$. $g(x) = \log(8) - \log(x+1)$.
What is/are the x-coordinate(s) of the intersection(s) of the graphs of f(x) and g(x) on the Cartesian plane?

a) {-3,3} b) {3} c) {-3} d) Ø e) NOTA

2. Completely simplify into one logarithm in the form $\log_a(b)$, where a and b are natural numbers and a is either 6 or 7, and give value of the sum of a and b:

$$\log_6(10)\log_7(9) + \log_6(4)\log_7(10)$$

a) 16 b) 17 c) 42 d) 107 e) NOTA

3. Evaluate: $\log_{12}(6) + \log_{12}(24)$.

a) $6\log_{12}(24)$ b) 2 c) 12 d) log(144) e) NOTA

4. Simplify: $i^{2004} + i^{2005} + i^{2006} + i^{2007}$, where $i = \sqrt{-1}$

a) 0 b) -2 c) $-2 - 2i$ d) $2 + 2i$ e) NOTA

5. Find the sum of the geometric series: $\pi + 1 + 1/\pi + 1/\pi^2 + 1/\pi^3 + \ldots$

a) $\frac{\pi}{\pi - 1}$ b) $\frac{\pi}{1 - \pi}$ c) $\frac{1}{\pi - 1}$ d) $\frac{\pi^2}{\pi - 1}$ e) NOTA

6. Evaluate: $\log_2(3) \cdot \log_3(4) \cdot \log_4(5) \cdot \log_5(6) \cdot \ldots \cdot \log_{126}(127) \cdot \log_{127}(128)$.

a) $\log_3(127)$ b) $\log_2(127)$ c) 3 d) 7 e) NOTA

7. Find the coefficient of the term containing $a^3b^2c^{10}d^2$ in the expansion of $(a+b+c+d)^{16}$.

a) 20,922,789,888,000 b) 348,713,164,800 c) 480,480
d) 240,240 e) NOTA

8. $f(x) = \log_2(x) + \log_3(x) + \log_4(x) + \log_5(x) + \ldots + \log_{100}(x)$. What is the x-intercept of the graph of f(x) on the Cartesian plane?

a) 0 b) log(2) c) 2 d) 100 e) NOTA

9. If f(x) = log(x), and f(x) = g(x), how many of the following expressions could be g(x)? c_1, c_2, c_3 and c_4 are real constants.

I. $\log(2006x) + c_1$ II. $c_2 \cdot \ln(x)$ III. $2006\log(x) + c_3$ IV. $\log_{C4}(x^{2006})$

a) 0 b) 1 c) 2 d) 3 e) NOTA

10. Put the following in order from least to greatest:

I. 2^{1000} II. 3^{800} III. 4^{750} IV. 5^{300}

a) IV, III, II, I b) I, II, III, IV c) IV, II, I, III
d) IV, I, II, III e) NOTA

11. Reduce $\sqrt{79+20\sqrt{3}}-\sqrt{28+6\sqrt{3}}$ to the form $A+B\sqrt{C}$ (in simplest radical form), where A, B and C are integers. Now, find A+B+C.

a) 9 b) 6 c) 3 d) 0 e) NOTA

12. $A = 2^{2005} - 2^{2004} + 2^{2003} - 2^{2002} + \ldots + 2^1 - 2^0$. Find log(A), to the nearest integer.

a) 602 b) 603 c) 604 d) 2005 e) NOTA

13. What is the remainder when $1^{2006} + 2^{2006} + 3^{2006} + \ldots 100^{2006}$ is divided by 10?

a) 0 b) 3 c) 5 d) 8 e) NOTA

14. The number of aardvarks varies jointly with the $\log_2$ of the number of bears and the $\log_3$ of the number of cats. When there are 16 bears and 3 cats, there are 16 aardvarks. How many bears are there if there are 27 cats and 12 aardvarks?

a) 2 b) 1 c) 1/2 d) 0 e) NOTA

15. What is the sum of all the coefficients in the expansion of $(a^2 + b^2 + c^2 + ... + y^2 + z^2)^2$

a) 26 b) 52 c) 325 d) 676 e) NOTA

16. Simplify: $\left(\sqrt[4]{2^6}\right)^2$

a) 2 b) $2\sqrt{2}$ c) 8 d) 16 e) NOTA

17. Approximate $(8+1)^{1/3}$ using the first three terms of the binomial expansion. (The first three terms are the ones with the powers $(8^{1/3})$, $(8^{-2/3})(1^1)$ and $(8^{-5/3})(1^2)$.)

a) $\frac{2082}{1001}$ b) $\frac{668}{321}$ c) $\frac{599}{288}$ d) $\frac{52}{25}$ e) NOTA

18. g(x) = antilog(x), where $g^{-1}(x) = \log(x)$. What is g(1)?

a) 0 b) 1 c) 10 d) 100 e) NOTA

19. Solve for x over the reals: $4^x + 2^x + 1^x = 73$

a) {-9,8} b) {8} c) {3} d) {-9,3} e) NOTA

20. Find the sum of the least nonnegative integers *a* and *b* which satisfy

$$\log_2(a) + \log_b(3) = 3$$
$$\log_a(2) + \log_3(b) = 3/2$$

a) 25 b) 7 c) 5 d) 1 e) NOTA

21. Find the principal value of i^i to the nearest hundredth, where $i = \sqrt{-1}$.

a) 4.81 b) 1.00 c) 0.21 d) 1.00*i* e) NOTA

22. 10g of a certain radioactive isotope, which decays exponentially, is found at noon, March 14th, 2006. At noon, March 14th, 2011, there is 8g of that substance. To the nearest thousandth of a year, what is the isotope's half-life?

a) 12.500 b) 13.425 c) 15.531 d) 67.126 e) NOTA

23. Find the value of $\sqrt{12-\sqrt{12-\sqrt{12-\sqrt{12-\sqrt{\cdots}}}}}$

a) 2 b) 3 c) $2\sqrt{3}$ d) 4 e)NOTA

24. Given that $x + \frac{1}{x} = n$, the summation $\sum_{k=-3}^{3} x^k$ can be expressed as a polynomial in terms of n (not x). Find the sum of the coefficients in that polynomial.

a) -2 b) -1 c) 0 d) 1 e) NOTA

25. If 0.123123123… is a base-4 number, what is its value in base-10?

a) $\frac{41}{333}$ b) $\frac{3}{7}$ c) $\frac{1}{5}$ d) $\frac{205}{666}$ e) NOTA

26. Evaluate: $|(6-8i)(4+3i)|$, where $i = \sqrt{-1}$.

a) $48 - 14i$ b) $48 + 14i$ c) 50 d) 21 e) NOTA

27. Solve for x: $7^{2x+1} = 7^{x-3}$

a) 3 b) 1 c) $\frac{1}{3}$ d) -4 e) NOTA

28. 2006! can be represented as $a \cdot 10^b$, where both a and b are integers. What is the maximum possible value for b?

a) 2006 b) 501 c) 500 d) 499 e) NOTA

29. If $x = 15_{HEX}$, $y = 1500_{HEX}$ and $z = x^y$ is a base-16 number, how many digits are in z's expansion (*e.g.* $2A^2_{HEX} = 6E4_{HEX}$ would have three digits)? Note that the subscript "HEX" indicates that a number is a hexadecimal (base-16) number.

a) 1466 b) 1765 c) 5789 d) 5904 e) NOTA

30. Approximate x to the nearest hundredth: $11^x = 82$.

a) 1.80 b) 1.81 c) 1.82 d) 1.83 e) NOTA