

Directions: For the purposes of this test, NOTA denotes None of the Above. Let $i = \sqrt{-1}$.

1. What is the y-intercept of the line passing through the points (1, 4) and (2, -2)?
 A. (0, -6) B. (0,10) C. (10,0) D. (-6, 0) E. NOTA

2. Find the positive difference between the sum of the first 10 positive even numbers and the sum of the first 10 positive odd numbers.
 A. 10 B. -10 C. 5 D. 1 E. NOTA

3. Find $b_1 + b_3 + b_5 + \dots + b_{15}$ given that $\sum_{i=1}^n b_i = n^2$ (that is, the sum of the first n terms is n^2).
 A. 120 B. 225 C. 105 D. 100 E. NOTA

4. Solve the inequality $\frac{3-x}{4} > \frac{1}{2+x}$, where both curves are defined. Give your answer in interval notation.
 A. $(-\infty, 2)$ B. $(-\infty, -2) \cup (-1, 2)$ C. $(-\infty, -1] \cup [2, \infty)$ D. $(-1, 2)$ E. NOTA

5. The function $p(x) = x^2 + ax + b$ has roots c and d , while the function $q(x) = x^2 + cx + d$ has roots a and b . Find $a + b + c + d$.
 A. 0 B. 1 C. 2 D. Cannot be determined E. NOTA

6. Evaluate $\frac{(1+i)^{10}}{(1-i)^{10}}$.
 A. $32i$ B. -1024 C. 32 D. -1 E. NOTA

7. Express the following sum in base ten: $1001_2 + 2002_3 + 3003_4$.
 A. 6006 B. 260 C. 210 D. 1337 E. NOTA

8. The ellipse $4x^2 + 9y^2 + 8x + 36y - 104 = 0$ has its center at (h, k) and foci at (a, b) and (c, d). Find $a + b + c + d + h + k$.
 A. $-9 + 4\sqrt{5}$ B. -3 C. -9 D. $4\sqrt{5}$ E. NOTA

9. Find the exact value of the following product: $(\log_{13} 25)(\log_4 12)(\log_5 64)(\log_{144} 169)$.
 A. $2\log 2$ B. 12 C. 1 D. 6 E. NOTA

10. In Mrs. Doker’s notoriously difficult Algebra II class, there are 12 desks (in a row) but only 8 students. What is the probability that the students sit such that no two adjacent desks are both empty? Assume that students choose to sit in desks at random.

- A. $\frac{1}{165}$ B. $\frac{14}{55}$ C. $\frac{1}{{}_{19}C_8}$ D. $\frac{1}{40320}$ E. NOTA

11. Solve for x: $x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}$.

- A. $\frac{1 + \sqrt{5}}{2}$ B. 1 C. $\sqrt{2}$ D. $\frac{1 - \sqrt{5}}{2}$ E. NOTA

12. Consider the following linear system of equations: $\begin{cases} 2x + 3y = k \\ x + py = p^2 \end{cases}$, where p and k are constants. Find the value of k that makes the system dependent.

- A. $\frac{3}{4}$ B. $\frac{9}{4}$ C. 4 D. 2 E. NOTA

13. The mathematical expression $j \equiv k \pmod n$ means that $(j - k)$ is an integer multiple of n . For how many *positive* integers n does $241 \equiv 1 \pmod n$ hold true?

- A. 20 B. 240 C. ∞ D. 19 E. NOTA

14. How many ordered sets of positive integers (w, x, y, z) satisfy $w + x + y + z = 12$?

- A. 4096 B. 455 C. 1024 D. 165 E. NOTA

15. A triangle is plotted in a coordinate plane. The triangle has vertices at the points $A(3, 4)$, $B(5, 1)$, and $C(1, 1)$. A circle is then circumscribed around this triangle. If the circle has center (h, k) and radius r , find $h + k + r$.

- A. 7 B. 3 C. 4 D. $\frac{13}{2}$ E. NOTA

16. If the roots of $g(x) = x^3 + 2x^2 + 2x + 1$ are r_1, r_2, r_3 , find the polynomial whose roots are $2r_1 - 2, 2r_2 - 2, 2r_3 - 2$.

- A. $f(x) = x^3 + 10x^2 + 36x + 48$ C. $f(x) = 8x^3 - 4x^2 + 2x$
 B. $f(x) = x^3 + 5x^2 + 9x + 6$ D. $f(x) = x^3 + 2x^2 + 4x + 8$ E. NOTA

17. Find the following sum: $i + 2i^2 + 3i^3 + \dots + 100i^{100}$.
 A. 0 B. $100i$ C. $-100+50i$ D. $50-50i$ E. NOTA
18. If the sum of all solutions to the equation $(x^{\log 3})^2 - 3^{\log x} - 2 = 0$ is $a^{\log_b c}$, find $a + b + c$, where b and c are prime numbers and a , b , and c are all positive integers.
 A. 15 B. 0 C. 1 D. 12 E. NOTA
19. Which has a greater area, a square of side length $\sqrt[10]{10}$ or a square of side length $\sqrt[3]{2}$?
 A. Side length $\sqrt[10]{10}$ B. Side length $\sqrt[3]{2}$ E. NOTA
 C. Both have equal area. D. Impossible to determine
20. A river flows at 6 mph, while a boat holding Murban Eyer, Carry Loker, Tim Jressel, and Bobby Bowden can travel 10 mph in still water. The boat is 8 miles upstream from their camp when Bobby Bowden falls out of the boat. Of course, nobody notices until they reach their camp. At that point, they immediately turn around and head back upstream. Assuming that Bobby Bowden floats downstream at the same rate as the current and no time is lost turning the boat around, how far (in miles) from their camp do they finally reach Bobby?
 A. 3 B. 4 C. 2 D. 1 E. NOTA
21. How many positive numbers less than or equal to 1,000 are divisible by 2 or 3?
 A. 667 B. 833 C. 666 D. 751 E. NOTA
22. Marshall and Alex are playing a game. Marshall starts with 8 chips and Alex with 6 chips. Alex rolls his fair die, and the number on top is the number of chips he takes from Marshall. Marshall then rolls his fair die, and the number on top is the number of chips he takes from Alex. Whoever has the most number of chips wins. If there is a tie, they roll another round (but then they both start with 7 chips). This process continues until a winner is determined. What is the probability Alex wins?
 A. $\frac{5}{18}$ B. $\frac{25}{72}$ C. $\frac{1}{3}$ D. $\frac{155}{432}$ E. NOTA
23. Thetis, who is currently at the point $(1, -2)$, needs to take Achilles to the river Styx (represented by the line $y = x + 1$) to give him his morning bath. She must then take him to the point $(2, 1)$, where he will be trained as a warrior. Find the shortest distance that Thetis can travel.
 A. $2\sqrt{17} + \frac{\sqrt{5}}{5}$ B. $2\sqrt{17}$ C. $\sqrt{26}$ D. 13 E. NOTA

24. Find the coefficient of the 3rd term of the expansion of $(3x - y)^5$.
 A. -90 B. 270 C. 10 D. -10 E. NOTA
25. Find $A - B + C$ if $\frac{x+1}{(x^2+1)(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1}$, where A, B, and C are constants.
 A. 0 B. -2 C. 2 D. 1 E. NOTA
26. What is the sum of the ordinates and abscissas of all intersection points between $f(x) = x^3 + 3x^2 + 2x - 1$ and its inverse relation?
 A. $\frac{1}{2}$ B. 6 C. -6 D. 0 E. NOTA
27. Melissa needs to fence in her cows (in a rectangular closure) to keep them from running away and jumping over the moon. If one side of the enclosure is her house of infinite length (this side needs no fence) and she has 200 feet of fence, what is the largest area (in ft^2) that she can enclose?
 A. 2,500 B. 2,000 C. 1,000 D. $1,000\pi$ E. NOTA
28. If two linear equations $\begin{matrix} ax + by = c \\ dx + ey = f \end{matrix}$ are solved with Cramer's rule, two of the three determinants encountered are $\begin{vmatrix} 4 & 6 \\ 2 & 3 \end{vmatrix}$ and $\begin{vmatrix} 4 & 1 \\ 2 & -1 \end{vmatrix}$. Given that this system has a unique solution (x, y) , what is $x + y$?
 A. Cannot be determined B. 1.5 C. 2 D. 0 E. NOTA
29. The length of the diagonal of a box is 5. The total surface area is 50. Find the sum of the lengths of all edges.
 A. 35 B. 10 C. $3\sqrt{5}$ D. $5\sqrt{3}$ E. NOTA
30. If $\sqrt{4 + 4i\sqrt{3}} = a + bi = z$, what is the magnitude of z ?
 A. 5 B. $2\sqrt{2}$ C. $4 + 4i\sqrt{3}$ D. 4 E. NOTA