

The abbreviation "NOTA" means "None of These Answers."

1. The function  $f$  is an **odd** function with domain all real numbers.

$x$	-2	-1	1	2
$f(x)$	4	0	$a$	$b$

Give the value of  $a - b$ .

- A. 0                      B. 4  
 C. -4                     D. 8  
 E. NOTA
2. After an increase of 10%, Sam's weekly pay was 20% of Ralph's. If Ralph's weekly pay was  $X$  dollars, then how many dollars was Sam's weekly pay before the increase?

- A.  $\frac{11}{2}X$                 B.  $2X$   
 C.  $\frac{1}{2}X$                 D.  $\frac{2}{11}X$   
 E. NOTA

3. If  $\cos \theta = \frac{a}{4}$  for  $0 < \theta < \frac{\pi}{2}$  then which is an expression for  $\tan \theta$ ?

- A.  $\frac{4-a}{a}$                     B.  $\frac{\sqrt{16-a^2}}{a}$   
 C.  $\frac{\sqrt{16-a^2}}{4}$                 D.  $\frac{4}{\sqrt{16+a^2}}$   
 E. NOTA

4. A regular hexagon has a diagonal chosen at random. What is the probability that this diagonal has the maximum length of the set of its diagonals?

- A.  $\frac{1}{3}$                       B.  $\frac{2}{9}$   
 C.  $\frac{1}{6}$                       D.  $\frac{1}{9}$   
 E. NOTA

5.  $\sqrt{r} + \sqrt{s} = 3$  and  $rs = \frac{1}{9}$ . If  $r$  and  $s$  are both positive real numbers then what is the value of  $\sqrt{r+s}$ ?

- A.  $\frac{25}{9}$                       B.  $\frac{5\sqrt{3}}{3}$   
 C.  $\frac{5}{3}$                       D.  $\frac{3\sqrt{3}}{4}$   
 E. NOTA

6. A parabola has directrix  $y = \frac{3}{4}x - 1$  and focus  $(-1, 4)$ . What is the length of its latus rectum (focal width)?

- A.  $\frac{46}{5}$                       B.  $\frac{23}{5}$   
 C.  $\frac{23}{10}$                      D.  $\frac{5}{23}$   
 E. NOTA

7.  $\triangle RST$  has sides  $RS = 6$ ,  $ST = \sqrt{39}$ , and  $RT = 3$ . What is the value of  $\cos R$ ?

A.  $\frac{1}{35}$       B.  $\frac{1}{13}$   
 C.  $\frac{1}{6}$       D.  $\frac{1}{3}$   
 E. NOTA

8. What is the period of the graph of  $f(x) = 3\cos^2(2x) - 3\sin^2(2x)$ ?

A.  $\pi$       B.  $\frac{\pi}{2}$   
 C.  $\frac{\pi}{3}$       D.  $\frac{\pi}{4}$   
 E. NOTA

9. How many positive integral factors of the expansion of  $100!$  are evenly divisible by 6?

A. 33      B. 34  
 C. 45      D. 48  
 E. NOTA

10. What is the middle term of the expansion of  $\left(\frac{1}{2}x^2 - 2\right)^6$ ?

A.  $-60x^4$       B.  $-20x^6$   
 C.  $20x^5$       D.  $60x^4$   
 E. NOTA

11. An unpainted wooden cube has each edge 4 cm and its surface is then painted blue on two opposite faces and white on the other faces. It is divided into 64 smaller cubes of edge lengths 1 cm. What fraction of the smaller cubes have one blue face and two white faces?

A.  $\frac{1}{16}$       B.  $\frac{1}{8}$   
 C.  $\frac{1}{4}$       D.  $\frac{1}{3}$   
 E. NOTA

12. For  $i = \sqrt{-1}$ ,  $(1 - i\sqrt{3})^4 = a + bi$ . Give the value of  $b$ .

A.  $-8\sqrt{3}$       B.  $-8$   
 C. 8      D.  $8\sqrt{3}$   
 E. NOTA

13. Which are the asymptotes of the graph defined by parametric equations

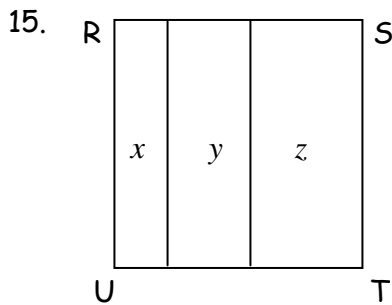
$$x(t) = \frac{1}{3-t} \text{ and } y(t) = \frac{1}{t}, \text{ graphed on the } xy\text{-plane?}$$

I.  $x = 3$   
 II.  $y = 0$   
 III.  $y = \frac{1}{3}$   
 IV.  $x = \frac{1}{3}$

A. I, II only      B. II, III only  
 C. II, IV only      D. III, IV only  
 E. NOTA

14. If  $a^2 - 3a - 4 < 0$  then what is the least integral value of  $a$ ?

- A. -1
- B. 0
- C. 1
- D. 4
- E. NOTA



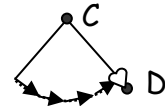
Square RSTU is divided into regions  $x$ ,  $y$  and  $z$  such that the areas of these regions are in an arithmetic progression. If  $RS=6$  then what is the area of region  $y$ ?

- A. 6
- B. 8
- C. 10
- D. 12
- E. NOTA

16.  $\log_3 \sqrt{3} + \log_3 \sqrt{9} + \log_3 \sqrt{27} =$

- A. 4
- B. 3
- C.  $\frac{3}{2}$
- D.  $\frac{3}{4}$
- E. NOTA

17. A pendulum is attached to a string that has a length of 10 cm and the pendulum itself has length 1 cm (CD below is 11 cm). On the first swing, the tip of the pendulum (D) covers an angle of  $120^\circ$ . On each subsequent swing, the pendulum travels  $\frac{3}{4}$  of the distance of the previous swing.



How far will the pendulum's tip travel, totally, in cm?

- A.  $\frac{88}{3}\pi$
- B.  $\frac{44}{3}\pi$
- C.  $11\pi$
- D.  $44\pi$
- E. NOTA

18. If  $\frac{3 + 3^2 + 3^3 + \dots + 3^{10}}{3^{-1} + 3^{-2} + 3^{-3} + \dots + 3^{-10}} = 3^x$  then give the value of  $x$ .

- A. 0
- B. 1
- C. 10
- D. 11
- E. NOTA

19. The base-five number  $1101_{five}$  is equivalent to a base-ten number  $K$ .  $K$  has how many positive integer factors?

- A. 2
- B. 3
- C. 6
- D. 8
- E. NOTA

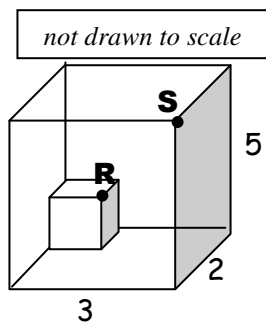
20. The term  $\cos \theta + \frac{1}{\cos \theta + \frac{1}{\cos \theta + \frac{1}{\dots}}}$  is equal to  $\frac{\sqrt{17} + 1}{4}$  for what value of  $\theta$  on the interval  $\left[0, \frac{\pi}{2}\right]$ ?

- A.  $\pi$                       B.  $\frac{\pi}{2}$   
 C.  $\frac{\pi}{3}$                       D.  $\frac{\pi}{6}$                       E. NOTA

21. The first of  $k$  consecutive integers is  $-7$ . The average of these  $k$  integers is  $\frac{15}{2}$ . Find the value of  $k$ .

- A. 10                      B. 20  
 C. 22                      D. 30                      E. NOTA

22. A large rectangular prism with dimensions 2 by 3 by 5 has a unit cube inside, so that they share one vertex. Three sides of the cube lie on three sides of the large prism. Find the distance from cube vertex R (which does not lie on the prism) to vertex S shown on the prism.



- A.  $\sqrt{3} - 1$                       B.  $\sqrt{7}$   
 C.  $\sqrt{15}$                       D.  $\sqrt{21}$   
 E. NOTA

23. A circle is defined by the polar equation  $r^2 - 8r \cos \theta + 6r \sin \theta = 0$  and is graphed on the  $xy$ -plane. The circle is the base of a cone with vertex at the point on the  $z$ -axis with rectangular coordinates  $(0, 0, 9)$ . Which is the volume of the cone?

- A.  $25\pi\sqrt{106}$                       B.  $225\pi$   
 C.  $36\pi\sqrt{34}$                       D.  $75\pi$   
 E. NOTA

24.  $\left| \frac{(1-i)^2}{3+4i} \right| =$

- A.  $\frac{1}{5}$                       B.  $\frac{2}{5}$   
 C.  $\frac{7}{5}$                       D. 5  
 E. NOTA

25. A car is driving up an incline with an acute angle with the horizontal that is  $\theta$ . If the car is traveling at 12 miles per hour and  $\tan \theta = \frac{4}{3}$  then how much time will it take the car to rise 6 miles?

- A.  $\frac{1}{6}$  hour                      B.  $\frac{5}{8}$  hour  
 C.  $\frac{2}{3}$  hour                      D.  $\frac{4}{5}$  hour  
 E. NOTA

26. If  $\sqrt{x^2 - 4x + 4} = 3x$  then find the sum of the real values of  $x$  which make the equation true.

- A.  $-\frac{1}{2}$       B.  $\frac{1}{2}$   
C.  $\frac{1}{4}$       D.  $\frac{1}{8}$   
E. NOTA

27. For  $\frac{1}{x} + \frac{1}{3} = \frac{2}{5}$ , give the value of  $x$ .

- A. 15      B. 10  
C. 3      D. 1  
E. NOTA

28. A snail is climbing out of a well that is 20 feet deep. She climbs 4 feet per day and then slides 1 foot back down at night (when it is dark) as she snoozes. (Silly snail shouldn't snooze). Another snail begins at the top of the well when the other snail is at the bottom. The "higher snail" slides a half foot downward at a constant rate each 24 hours. If the "start position" is the beginning of the first day when one snail is on top of the well, and the other on the bottom, during which time interval do the snails pass each other?

- A. On the 6<sup>th</sup> day, before dark.  
B. On the 6<sup>th</sup> day, after dark.  
C. On the 7<sup>th</sup> day, before dark.  
D. On the 7<sup>th</sup> day, after dark.  
E. NOTA

29. An even number of children are standing in a circle. They are equally spaced, and the 70<sup>th</sup> child is directly opposite the 180<sup>th</sup> child. How many children are in the circle?

- A. 221      B. 220  
C. 119      D. 118  
E. NOTA

30. A hyperbola has equation  $4x^2 - y^2 = 4$  and a rectangle has its vertices on the endpoints of the latus rectum of the graph. If the area of the rectangle is  $B\sqrt{C}$  for  $C$  having no perfect square factors larger than 1, then give the value of  $B + C$ .

- A. 13      B. 20  
C. 21      D. 23  
E. NOTA