For all questions, answer *E. NOTA* should be interpreted as *None of the above*.

$\mathbb{N}, \mathbb{Q}, \& \mathbb{R}$ represent the sets of natural, rational, and real numbers, respectively. $\mathbb{Z}$ is the set of integers.

For Questions 1-5, consider the functions

$$f(x) = \frac{1}{x} \text{ and } g(x) = x^2 + 3x + 2.$$

1. Find $f\big(g(2)\big) - g\big(f(2)\big)$.
   A. 0
   B. $\frac{11}{3}$
   C. $-\frac{11}{3}$
   D. Undefined
   E. NOTA

2. How many of the following statements are true?

   I. $f(x)$ is defined on the interval $[-1, 4]$.
   II. $g(x)$ is continuous on the interval $[-1, 4]$.
   III. $f(g(x))$ is defined at $x = 0$.
   IV. $g\big(f(x)\big)$ is defined at $x = 0$.

   A. 1
   B. 2
   C. 3
   D. 4
   E. NOTA

3. What is the range of $g(x)$ on the interval $[-4, 4]$?
   A. $[6, 30]$
   B. $[6, \infty)$
   C. $\left[-\frac{1}{4}, 30\right]$
   D. $[-\frac{1}{4}, \infty)$
   E. NOTA

4. Find $g\Big(f\big(f(g(3))\big)\Big)$.
   A. -3
   B. 3
   C. $\frac{1}{3}$
   D. $-\frac{1}{3}$
   E. NOTA

5. Find $f(1) + f(2) + f(4) + f(8) + \cdots$
   A. 1
   B. 2
   C. 4
   D. ∞
   E. NOTA

For Questions 6-7, consider the following function:

$$f(x) = \frac{x^2 - 3x + 2}{x^2 + x - 2}$$

6. How many asymptotes does $f(x)$ have?
   A. 0
   B. 1
   C. 2
   D. 3
   E. NOTA

7. At $x = 2$, $f(x)$
   A. is nonzero
   B. is zero
   C. has an asymptote
   D. has a hole in the graph
   E. NOTA

For Questions 8-10, consider the following function:

$$f(x) = 2x^7 - 6x^6 + x^5 - x^4 + 4x^3 + 2x^2 - x + 12$$

8. Find the sum of the roots of $f(x)$.
   A. 3
   B. -3
   C. 6
   D. -6
   E. NOTA

9. Given that all of the roots of $f(x)$ are distinct, how many are non-real?
   A. 1
   B. 3
   C. 5
   D. 7
   E. NOTA

10. How many of the following are true?

    I. $f(x)$ is an odd function
    II. $f(x)$ has a real root
    III. $f(x)$ has a complex root
    IV. $f(x)$ has degree 7

    A. 1
    B. 2
    C. 3
    D. 4
    E. NOTA

For Questions 11-13, consider the function defined by:

$$f(x) = \begin{cases} 1, & \text{if } x = 1; \\ 1, & \text{if } x = 2; \\ f(x-2) + f(x-1), & \text{if } x \geq 3. \end{cases}$$

11. What is the domain of $f(x)$?

A. $\mathbb{N}$
B. $\mathbb{Q}$
C. $\mathbb{R}$
D. $\mathbb{Z}$
E. NOTA

12. Find $f(5) - f(4) + f(3) - f(2) + f(1)$.

A. 4
B. 5
C. 12
D. 15
E. NOTA

13. Which of the following is equivalent to $f(10)$?

A. $\sum_{i=1}^{8} f(i)$
B. $\sum_{i=1}^{8} f(i) + 1$
C. $\sum_{i=1}^{8} f(i+1)$
D. $\sum_{i=1}^{8} f(i) + 8$
E. NOTA

14. Find the remainder when f is divided by $(x+1)$ if $f(x) = 2008x^{2008} + 2007x^{2007} + \cdots 2x^2 + x$.

A. 0
B. 2008
C. $\frac{(2009)(2008)}{2}$
D. 1004
E. NOTA

For Questions 15-16, consider the functions

$$f(x) = |x-3| - 1 \text{ and } g(x) = -|x-3| + 3.$$

15. f and g intersect at (a, b) and (c, d). Find $abcd$.

A. 5
B. 8
C. 10
D. 27
E. NOTA

16. Find the area enclosed by f and g.

A. 4
B. $4\sqrt{2}$
C. 8
D. 16
E. NOTA

17. Let $f(x) = Ax^3 + Bx^2 + Cx + D$ contain the points $(-3, 5), (-2, -6)$, and $(1,13)$. Find $A + B + C + D$.

A. 4
B. 5
C. 10
D. 13
E. NOTA

18. Let $y = ax^2 + bx + c$ be the parabola with focus at $(2, 1)$ and directrix $y = 3$. Find $\frac{bc}{a}$.

A. 2
B. -2
C. 4
D. -4
E. NOTA

19. Which of the following is true of $f(x) = e^x$?

A. The range of f is $(-\infty, \infty)$.
B. $f^{-1} = f$
C. f is one-to-one.
D. The domain of f is $\mathbb{N}$ and the range is $\mathbb{R}$.
E. NOTA

20. Which of the following is **false** for odd functions f and g?

A. $-f(a) = f(-a)$
B. f is symmetric with respect to the origin.
C. f cannot be even.
D. If $h(x) = f(x) + g(x)$, then $h(x)$ is odd.
E. NOTA

21. Which of the following relations **might not** be functions of x, if $f(x)$ and $g(x)$ **are** functions?

A. $h(x) = |f(x)|$
B. $h(x) = f(x) + g(x)$
C. $h(x) = f(x) \cdot g(x)$
D. $h(x) = f(g(x))$
E. NOTA

22. Which quadrants contain the function of y $f(y) = y^2 + 4$?

A. I only
B. I and II only
C. I and IV only
D. III and IV only
E. NOTA

23. Which of the following relations is a function of x?

A. $\sqrt{x} = \sqrt{y}$
B. $x^2 = y^2$
C. $|x| = |y|$
D. $c^x = c^y$, for all $c \in \mathbb{R}$
E. NOTA

24. If $g(x) = x^2$ and $f\big(g(x)\big) = \log x$, find $f(100)$.

A. 0
B. 1
C. 2
D. 10,000
E. NOTA

25. Given $f(x) = \frac{1}{x}$, solve for x where $f(x) = f^{-1}(x)$.

A. $\{0\}$
B. $\{1\}$
C. $\{\pm 1\}$
D. $\mathbb{R}$
E. NOTA

26. If $f(x) = \frac{1}{x+1}$, find $[f(x)]^2 - f(x^2)$ where defined.

A. 0
B. $\frac{1}{2x}$
C. $\frac{2x}{(x+1)^2(x^2+1)}$
D. $-\frac{2x}{(x+1)^2(x^2+1)}$
E. NOTA

27. Let $f(x)$ be an exponential function of the form $f(x) = a \cdot b^x$ such that $f(0) = 4$ and $f(3) = 108$.

Find $f(2) - f(1)$.

A. 3
B. 6
C. 9
D. 24
E. NOTA

28. Find the product of the complex solutions to:

$$f(x) = 3x^3 - 2x^2 + 12x - 8$$

A. 4
B. -4
C. $\frac{2}{3}$
D. $\frac{8}{3}$
E. NOTA

29. Let $f(x) = 1 + \frac{1}{x}$. Find all x where $x = f(x)$.

A. $x = 1$
B. $x = 2$
C. $x = \frac{1+\sqrt{5}}{2}$
D. $x = \frac{1\pm\sqrt{5}}{2}$
E. NOTA

30. An open box is formed by cutting out squares of length s from the corners of a rectangular piece of cardboard with dimensions x and y. Express the volume of the box as a function of s, x, and y, where s, x, and y give a positive real volume.

A. $V(s,x,y) = sxy$
B. $V(s,x,y) = s(x-s)(y-s)$
C. $V(s,x,y) = s(x-2s)(y-2s)$
D. $V(s,x,y) = 2s(x-2s)(y-2s)$
E. NOTA