

Mu Alpha Theta National Convention 2007
Sequences and Series Alpha

For all questions, E. NOTA means none of the above answers is correct.
 Calculators are not permitted. S_n represents the n^{th} term of a sequence S .

1. What is the common difference of the arithmetic sequence 10, 36, 62, 88, ...?

- A. 3.6 B. 26 C. 28 D. 78 E. NOTA

2. What is the common ratio of the geometric sequence $\frac{1}{2}, \frac{\pi}{2}, \frac{\pi^2}{2}, \dots$?

- A. π B. $\frac{\pi}{\sqrt{2}}$ C. $\frac{\pi}{2}$ D. $\frac{1}{2}$ E. NOTA

3. The first three terms of the sequence $a_n = \frac{2n}{n+2}$ are $\frac{2}{3}, 1, \frac{6}{5}, \dots$. What is the next term?

- A. $\frac{7}{9}$ B. $\frac{4}{3}$ C. $\frac{12}{7}$ D. 3 E. NOTA

4. Given the sequence $b_n = \sin\left(\frac{n \cdot \pi}{2}\right)$, what is b_3 ?

- A. -1 B. $-\frac{\sqrt{2}}{2}$ C. 0 D. $\frac{\sqrt{2}}{2}$ E. NOTA

5. The first 10 terms of the sequence $\{c_n\}$ are 1, 7, 25, 79, 241, 727, 2185, 6559, 19681, 59047.
 What is the general formula for c_n ?

- A. $5n - 2$ B. $\frac{3^n - 2}{3}$ C. $3^n - 2$ D. $n^3 + 5$ E. NOTA

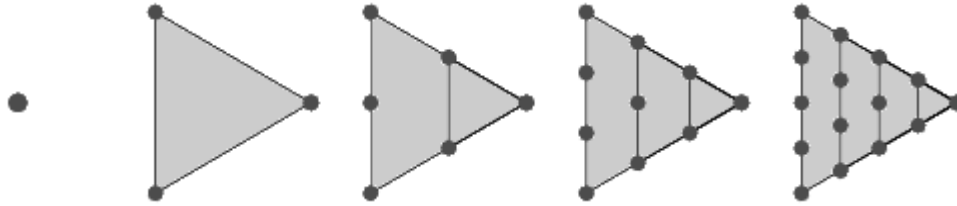
6. Evaluate: $\sqrt{30 + \sqrt{30 + \sqrt{30 + \dots}}}$

- A. $2\sqrt{30}$ B. $\sqrt{30}i$ C. 6 D. 7 E. NOTA

7. Evaluate: $\sum_{k=4}^9 (5k^2 - 2k + 1)$

- A. 279 B. 1277 C. 1283 D. 1439 E. NOTA

8. The triangular number T_n is a number that can be represented in the form of a triangular grid of points where the first row contains a single element and each subsequent row contains one more element than the previous one. This is illustrated for $T_1 = 1$, $T_2 = 3$,



What is the sum of the first seven triangular numbers?

- A. 28 B. 56 C. 64 D. 84 E. NOTA

9. Given that $a_1 = 3$ and $a_{k+1} = 2(a_k - 1)$, find $a_6 - a_3$.

- A. 15 B. 24 C. 27 D. 30 E. NOTA

10. Given that $a_1 = 3$ and $a_{k+1} = 3a_k - 2$, the general equation for a_k is written in the form $a_k = A(B^{k-1}) + C$. What is $A + B - C$?

- A. 0 B. 4 C. 6 D. 7 E. NOTA

11. Nate drops a racquetball from a height of 15 meters. On each bounce, the ball reaches one-third of the height from which it previously fell. What is the total vertical distance the ball travels from the time it is dropped to the time it comes to rest?

- A. 25.5 m B. 30 m C. 35 m D. 45 m E. NOTA

12. Evaluate: $2 \cdot \sum_{k=1}^{15} \sin\left(\frac{\pi \cdot k}{4}\right) \cos\left(\frac{\pi \cdot k}{4}\right)$

- A. -1 B. 0 C. 1 D. $\frac{\sqrt{2}}{2}$ E. NOTA

13. Evaluate: $1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \dots}}}}$

- A. $\frac{1+\sqrt{3}}{2}$ B. $\frac{1+\sqrt{21}}{6}$ C. $\frac{3+\sqrt{21}}{6}$ D. $1 + \frac{1}{\sqrt{3}}$ E. NOTA

14. ^{52}Fe is a key isotope of iron that has a half-life of 8 hours. Twelve hours after John began an experiment with K grams of the isotope, he determined that there were 50 grams left. What is the value of K in grams?

- A. $50\sqrt{2}$ B. $80\sqrt{2}$ C. 140 D. 200 E. NOTA

15. What are the last two digits of 21^{100} ?

- A. 41 B. 21 C. 01 D. 00 E. NOTA

16. What is the next number in the sequence 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...?

- A. 45 B. 47 C. 55 D. 65 E. NOTA

17. Each of the solid rocket boosters (SRB) on the space shuttle uses 55% of its remaining fuel every 30 seconds after liftoff. The propellant for each solid rocket motor weighs approximately 1,100,000 pounds (499,000 kg) prior to ignition. How much time in seconds has elapsed when each of the SRBs have 49,900 kg of propellant left?

- A. $30 \cdot \frac{\log(0.55)}{\log(0.1)}$ B. $30 \cdot \frac{\log(0.1)}{\log(0.45)}$ C. $55 \cdot \frac{\log(0.3)}{\log(0.1)}$
 D. $30 \cdot \frac{\log(0.1)}{\log(0.45)} + 30$ E. NOTA

18. The sides of a right triangle form a geometric progression. Given that θ is the smallest angle in the triangle, what is $\sin(\theta)$?

- A. $\frac{1+\sqrt{5}}{4}$ B. $\frac{\sqrt{5}-1}{2}$ C. $\sqrt{\frac{1+\sqrt{5}}{2}}$ D. $\sqrt{\frac{\sqrt{5}-1}{2}}$ E. NOTA

19. A series of matrices is produced according to the equation $X_n = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} \cdot \begin{bmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{bmatrix}^n$.

What is $|X_3|$?

- A. $(x+y+z)(a^2+b^2+c^2)$ B. 0 C. $3a+2b+c$
 D. 1 E. NOTA

20. What is the coefficient of $x^{12}y^{13}$ in the expansion of $(x+y)^{25}$?

- A. $\frac{25!}{13!}$ B. $\frac{25!}{12!}$ C. $\frac{26!}{13!12!}$ D. $\frac{25!}{13!12!}$ E. NOTA

21. Evaluate: $\sum_{n=2}^{500} (n \cdot n!)$

- A. $501! - 2$ B. $501! - 1$ C. $500! + 1$ D. $501!$ E. NOTA

22. Evaluate: $\sum_{n=1}^{24} \frac{1}{\sqrt{n+1} + \sqrt{n}}$

- A. $6\sqrt{6}$ B. 7 C. 5 D. 4 E. NOTA

23. Which of the following are true?

- I. The sum of the first n positive integers is n^2
 II. $n < 2^n$ for all positive integers n
 III. $1 + 2 + 2^2 + \dots + 2^n = 2^{n+1} - 1$
 IV. $\prod_{j=0}^4 j! = 144$

- A. I only B. I, II, III only C. II, IV only
 D. III, IV only E. NOTA

24. Evaluate: $\sum_{k=1}^{16} \frac{1}{k(k+1)}$

- A. $\frac{8}{9}$ B. $\frac{16}{17}$ C. $\frac{23}{25}$ D. 1 E. NOTA

25. Considering the sequence $a_n = \frac{10^n}{n!}$ and its partial sum S_n , how many of the following are true?

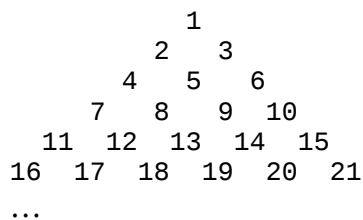
- I. The sequence is geometric for $n > 10$
 II. $S_3 > S_2 > S_1$
 III. There are two consecutive terms that are equal in value.

- A. I, III only B. II, III only C. II only D. III only E. NOTA

26. Evaluate: $\sum_{k=1}^{\infty} \frac{2}{k^2 + 2k}$

- A. $\frac{1}{2}$ B. $\frac{3}{4}$ C. $\frac{3}{2}$ D. 2 E. NOTA

27. Suppose that S_n gives the sum of the elements in the n^{th} row of the following triangle:



What is $S_9 - S_8$?

- A. 113 B. 119 C. 109 D. 85 E. NOTA

28. Given that a_n and b_n are two arithmetic sequences and A_n and B_n are sums of the first n elements. If $(4n + 27) \cdot A_n = (7n + 1) \cdot B_n$, what is $\frac{a_{11}}{b_{11}}$?

- A. $\frac{18}{7}$ B. $\frac{4}{3}$ C. $\frac{3}{4}$ D. $\frac{4}{7}$ E. NOTA

29. How many numbers between 4000 and 5000 are divisible by 9?

- A. 117 B. 111 C. 110 D. 109 E. NOTA

30. Given the function $f(x) = x^{x^{x^{x^{\dots}}}}$, solve for n where $f(n) = 2$.

- A. $\sqrt{2}$ B. $\frac{1}{\sqrt{2}}$ C. $\sqrt[4]{2}$ D. $\sqrt{3}$ E. NOTA