1) A
Since $r^2 = x^2 + y^2$ and $\sin(\theta) = \frac{x}{r}$, multiplying the equation by leads to the equation of a CIRCLE.

2) A
Using either integration by parts of tabular method leads to $\int = (x^2/2 - 2x/2 + 1/4)e^{2x}$ Evaluating from 2 to 1 leads to $5e^4/4 - e^2/4$.

3)C
The given function behaves enough like a rational function that its radius of convergence is the distance from where the series is centered to the first value that makes the function undefined. This occurs at $x = -3$. So the radius of convergence is equal to 3.

4) D
$\arctan x = x - x^3/3 + x^5/5$... We know from Taylor, that the coefficent of the xth term is equal to the xth derivative divided by x! From this we can see that the 5th derivative of arctanx $= 5!/5 = 24$

5) A

$$V = \pi \int_1^{'\infty'} \frac{1}{x^2}\, dx = \pi$$

6) C

This equals $\int 1\, dx = x$ from 5 to 1 is 4.

7) B

$r = cos 2\theta$

$$\begin{aligned} A &= 2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2} r^2\, dr \\ &= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} cos^2 2\theta\, dr \\ &= 2\int_{0}^{\frac{\pi}{4}} cos^2 2\theta\, dr \\ &= \int_{0}^{\frac{\pi}{4}} 1 + cos 4\theta\, dr \\ &= \theta + \frac{1}{4}\sin 4\theta |_0^{\frac{\pi}{4}} \\ &= \frac{\pi}{4} \end{aligned}$$

8) A

$\frac{d^2y}{dx^2} = \frac{\frac{d\frac{dy}{dx}}{dt}}{\frac{dx}{dt}}$
$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$
$\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = 2$, thus $\frac{dy}{dx} = \frac{1}{t}$, $\frac{dy}{dx}dt = \frac{-1}{t^2}$.
Thus, $\frac{d^2y}{dx^2} = \frac{-1}{2t^3}$

9) D

I) Consider a curve that loops on itself. At the point where it crosses itself, the derivative is different depending if you come from the positive or the negative direction. Also note, even curves with sharp points don't necessarily have derivatives at those points. FALSE
II) This is false. Look at question 5.

10) C

Using u substition, with $u = \sqrt{x}$ and then integration by parts will lead to $-2(\sqrt{x}\cos\sqrt{x} - \sin\sqrt{x})$

11) B

We know that:

$$\begin{aligned} \frac{1}{1+x} &= 1 - x + x^2 - x^3 ... \\ x\frac{d\frac{1}{1+x}}{dx} &= x(-1 + 2x - 3x^2 ...) \\ \frac{-x}{(1+x)^2} &= -x + 2x^2 - 3x^3 ... \end{aligned}$$

Now plugging in $x = \frac{1}{3}$ leads to $-\frac{3}{16} = -\frac{1}{3} + \frac{2}{9} - \frac{1}{9} + \frac{4}{81} ...$

12) D

Through the method of partial fractions we can break up the integrand into $\frac{-1x}{5(x^2+1)} + \frac{2}{5(x^2+1)} + \frac{1}{5(x+2)}$ see that the integral is equal to $.2(\ln(x+2) - .5\ln(x^2+1) + 2\arctan x)$ which when evaluated from 0 to 1 leads to $-.3\ln 2 + .1\pi + .2\ln 3$. Therefor $a = .2, b = .1, c = -.3, a + b + c = 0$

13) C

$\frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$. Plugging in $r = 1 + sin\theta$ and then $\theta = \frac{\pi}{3}$ leads to the slope being -1.

14) A

The amplitude is nothing more than the maximum value of the function. Therefore taking the derivative and setting equal to 0 leads to $\tan x = \frac{4}{19}$. Plugging this into the original equation and using some basic trig facts leads to square of the amplitude being 377.

15) B

$A = \int_0^1 e^x - x\,dx = e - \frac{3}{2}$

16) E

This integral passes through a discontinuity at $x = \pi/2$ and the antiderivative is also undefined there. Therefore, this improper integral does not converge.

17) E

This function is even and therefore has coefficents of 0 for every x^n term where n is odd.

18) B

Average value is equal to $\dfrac{\pi \int_1^5 r^2\,dr}{5-1}$ This value equals B.

19) D

Implicit differentiation:

$$\begin{aligned} x^2 + y^2 &= (2x^2 + 2y^2 - x)^2 \\ 2x + 2yy' &= 2(2x^2 + 2y - x)(4x + 4yy' - 1) \end{aligned}$$

Now plugging in (0,.5) leads to $y' = 2y' - 1, y' = 1$

20) C

It is clear that the argument of the log goes to 2 since $e^{1/x}$ goes to 0. Thus, the overall limit is log2.

21) D

The triangle has area equal to 3 and has a centroid at (2/3,8/3). Using Pappus Theorem, the volume of the new solid will be $2\pi(3)(7 - 2/3) = 38\pi$.

22) C

Intersect at $3\sin\theta = 1 + \sin\theta, \sin\theta = .5, \theta = \pi/6$ and $5\pi/6$.

Symmetric about $\theta = \pi/2, A = .5\int_{\pi/6}^{5\pi/6}(3sin\theta)^2\,d\theta - .5\int_{\pi/6}^{5\pi/6}(1 + sin\theta)^2\,d\theta$

$$\begin{aligned} A &= 2(.5\int_{\pi/6}^{\pi/2} 9\sin^2\theta\, d\theta - .5\int_{\pi/6}^{\pi/2} 1 + 2sin\theta + \sin^2\theta\, d\theta) \\ &= \int_{\pi/6}^{\pi/2} 8\sin^2\theta - 1 - 2\sin\theta\, d\theta \end{aligned}$$

23) B

$$\begin{aligned} L &= \int_0^{2\pi} \sqrt{(x')^2 + (y')^2}\, d\theta \\ &= 2\int_0^{2\pi} \sqrt{(2(1-\cos\theta)^2 + (2\sin\theta)^2}\, d\theta \\ &= \int_0^{2\pi} \sqrt{4(1 - 2\cos\theta + \cos^2\theta + \sin^2\theta)}\, d\theta \\ &= 2\int_0^{2\pi} \sqrt{2(1-\cos\theta)}\, d\theta \end{aligned}$$

24) B

Using trig substitution leads to $-\dfrac{\sqrt{9-x^2}}{x} - \arcsin\dfrac{x}{3} + C$.

Therefore $A = -1, B = 9, C = -1, D = 2$, and $E = 1$.
So $A + B + C + D + E = -1 + 9 - 1 + 2 + 1 = 10$

25)B

In Sigma notation, this equals $\dfrac{1}{n}\displaystyle\sum_{i=1}^{n} f(\frac{i}{n}) = \int_0^1 f(x)\, dx = \int_0^1 \sqrt{x} = \frac{2}{3}$.

26) B

$$\begin{aligned} y' + 3x^2y &= 6x^2 \\ e^{x^3}y' + e^{x^3}y &= 6x^2e^{x^3} \\ \text{(using product rule backwards) } (ye^{x^3})' &= 6x^2e^{x^3} \\ \text{(integrating) } ye^{x^3} &= 2e^{x^3} + C \\ y &= 2 + \frac{C}{e^{x^3}} \end{aligned}$$

27) D

$\cos 3x \sin x = .5(\sin 4x - \sin 2x)$, So we just have to integrate the left hand expression to get $-1/8(\cos 4x - 2\cos 2x)$ which when evaluated from $\pi/6$ to 0 is $1/16$.

28) C

$|x| < 4$, $\frac{x}{4} < 1$. Also, $\dfrac{4}{4+x} = \dfrac{1}{1-(-\frac{x}{4})}$

Now by the geometric sum formula: $\dfrac{1}{1-(-\frac{x}{4})} = 1 - \dfrac{x}{4} + \dfrac{x^2}{16}...$

$\sum_{n=0}^{\infty}(-\frac{x}{4})^n$

29) C

I) True since an asymptote is just the behavior of a function as a variable goes to infinity, it can cross it somewhere earlier.
II) This is true by definition

30) E

Since x is being measured in degrees and not radians, $\int \cos x = \frac{180}{\pi}\sin x$. Therefore, the answer is .5.