

1. **C.** By the Second Derivative Test, if  $f$  has a horizontal tangent line, then the concavity determines the feature of the graph. If the graph is concave up at a horizontal tangent line, then the graph feature is a relative minimum.

2. **B.** Average value =  $\frac{1}{b-a} \int_a^b f(x)dx$ . Here, the integral is  $\frac{1}{4} \int_{-2}^2 x^2 dx = \frac{8/3 - (-8/3)}{4} = \frac{4}{3}$

3. **D.** The factorial function grows much faster than any exponential so eventually  $n!$  dominates  $e^{n^2}$ . Therefore, the limit is  $-\infty$ .

4. **C.** First we need to determine when  $f(x)$  is increasing.  $f'(x) = 3 - 3x^2 \rightarrow x = \pm 1$ . So  $g(x)$  gives us a rectangle with width 2 and height 1. Area = 2.

5. **B.** The maximum area of a triangle with a fixed perimeter occurs when the triangle is equilateral. Thus the side length is  $\frac{2s}{3}$ . Since the area of an equilateral triangle with side length  $x$  is given by  $\frac{x^2\sqrt{3}}{4}$ , our desired area is  $\frac{s^2\sqrt{3}}{9}$ .

6. **E.**  $f(x) = \ln x^{-1} = -\ln x$ . So  $f'(x) = \frac{-1}{x}$ . Thus,  $f'(1) = -1$ .

7. **C.** The Intermediate Value Theorem guarantees a root of  $f(x)$  when there exists points  $a$  and  $b$  such that  $f(a) < 0 < f(b)$  or  $f(a) > 0 > f(b)$  AND  $f(x)$  is continuous. Since continuity is implied by differentiability, conditions I and III suffice.

8. **D.**  $\int_{-2}^2 f(x)dx = 7 \rightarrow \int_2^{-2} f(x)dx = -7$ .  $\int_5^2 f(x)dx + \int_2^{-2} f(x)dx = \int_5^{-2} f(x)dx = (3) + (-7) = -4$ . Thus,  $\int_5^{-2} 2f(x)dx = -8$  and  $\int_5^{-2} -3dx = 21$ . Thus the answer is  $-8 + 21 = 13$ .

9. **D.** Suppose the lines are tangent to  $f(x)$  and  $g(x)$  at  $x = a$  and  $x = b$  respectively. Then because the tangent lines must have the same slope,  $f'(a) = f'(b) \rightarrow 2a - 2 = -2b + 10 \rightarrow a + b = 6$ . Suppose also that the tangent line is of the form  $y = mx + h$ . Then the points  $(a, f(a))$  and  $(b, f(b))$  must be on this line. Thus we have  $a^2 - 2ab + 3 = (2a - 2)(a) + h$  and  $-b^2 + 10b - 17 = (-2b + 10)(b) + h$ . Solving for  $h$  in both equations and setting equal yields  $a^2 + b^2 = 20$ . With this equation and  $a + b = 6$ , we have that either  $a = 2, b = 4$  or  $a = 4, b = 2$ . When  $a = 4$ , this yields the line with the larger slope:  $y = 6x - 13$ .

10. **E.**  $f'(x) = 6x^2 - 14x + 4 = 0 \rightarrow 3x^2 - 7x + 2 = 0 \rightarrow (3x - 1)(x - 2) = 0$ . Testing points on intervals shows that  $x = -1/3$  is a maximum and  $x = 2$  is a minimum.

11. **D.** The derivative of  $a^x$  is  $a^x \ln(x)$ . So  $f'(x) = \pi^{x^2} \ln \pi \cdot 2x$ .

12. **A.**  $h'(x) = g'(f(x) + 3x) \cdot (f'(x) + 3)$ . Thus  $h'(2) = g'(f(2) + 6) \cdot (f'(2) + 3) = g'(3) \cdot (-1 + 3) =$

$$(-2)(2) = -4$$

13. **B.** It is not necessary to integrate the function because it is odd on equal but opposite bounds and therefore the integral is 0.

14. **B.** The appropriate substitution to make is  $(x - 2) = 4 \sin(u)$ . Thus,  $dx = 4 \cos(u) du$ . So the new integral becomes  $\int_0^{\frac{\pi}{6}} \frac{16 \sin^2(u) \cdot 4 \cos(u) du}{16 \cos(u)} = \int_0^{\frac{\pi}{6}} 4 \sin^2(u) du = 2u - \sin 2u$  from  $u = 0$  and  $u = \frac{\pi}{6}$ . Evaluating gives  $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$

15. **C.** Make the  $u$ -sub  $u = x^3 + 2x$ . Thus  $du = 3x^2 + 2$ . The integral has the form  $\int_3^{33} \frac{2du}{u} = 2(\ln(33) - \ln(3)) = 2 \ln(11)$

16. **A.** The acceleration of the particle is given by  $s''(t) = 12t - 12 = 0 \rightarrow t = 1$ . The velocity at  $t = 1$  is given by  $s'(1) = 6 - 12 + 4 = -2$ .

17. **C.** Because each piece is continuous, we can plug in  $x = 2$  for both pieces and since the values are the same, the limit exists. Because  $f(2) = 0$ , the same as the limit,  $f(x)$  is continuous at 3. However, the derivatives  $3x^2$  and 1 do not match at  $x = 2$ , so it is not differentiable.

18. **D.**  $f(0) = \frac{A}{1+B} = 30$ . Using this, we can plug into  $f'(0) = \frac{A(30)B}{(1+B)} = 9 \rightarrow \frac{12B}{1+B} = 9 \rightarrow B = 3$  and  $A = 120$ . So as  $t \rightarrow \infty$ , we get that  $f(t) = 120$ .

19. **E.** We can use L'Hospitals rule on each of these limits.  $A = 1, B = \frac{-\pi}{3}, C = \frac{1}{2}$

20. **D.** Since  $y = x^3$ , we know that  $x = y^{1/3}$ . Thus,  $V = \pi \int_0^h (y^{2/3} dy) \rightarrow \frac{dV}{dt} = \frac{dV}{dh} \frac{dh}{dt} = (\pi h^{2/3}) \left( \frac{dh}{dt} \right)$ . We're given  $dV/dt$  and  $h$ , so using  $\frac{dV}{dt} = \frac{dV}{dh} \frac{dh}{dt} \rightarrow 32\pi = (4\pi) \left( \frac{dh}{dt} \right)$

21. **B.** Consider  $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$ . In order for the series to converge, this expression must be less than

1. Thus  $-1 < \frac{x}{\pi^2} < 1$  The endpoints cannot be included because both  $1 + 1 + 1 + \dots$  and  $1 - 1 + 1 - \dots$  diverge.

22. **B.** The equation of the line tangent to this curve is  $y = x$ . Since  $y = x$  outgrows  $y = e^{-x} \sin(x)$  when  $x > 0$ , we need only to consider when  $x < 0$ . When  $x = -\frac{\pi}{2}$ ,  $e^{-x} \sin x$  is below  $y = x$  which was already tangent at  $x = 0$ , but since  $y = e^{-x} \sin(x)$  has to come back up to  $y = 0$  at  $x = -2\pi$ , it will intersect once more.

